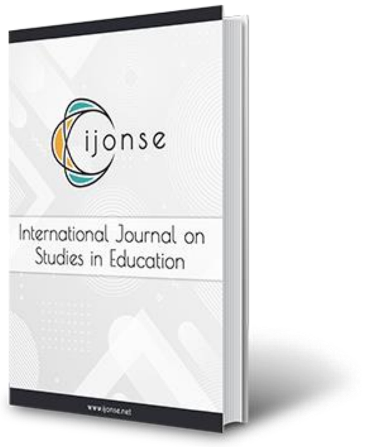

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Ellenita Garcia Manalaysay
Bulacan State University, Philippines

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Students' Engagement in a Mathematical Investigation through Online Problem-Based Learning

Ellenita Garcia Manalaysay

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Abstract

The core components of mathematical activity are involved in mathematical investigation: problem formulation, processing of conditions for which there is no clear solution, formulation and justification of conjectures, and generalization. However, introducing mathematical investigation to students is a challenging activity. A crucial aspect of the student's learning and understanding of the process is the kind and degree of their engagement in the learning activity. This phenomenological study sought to describe how pre-service teachers engaged in an online problem-based learning (PBL) while doing a mathematical investigation (MI). The researcher explored the cognitive, behavioral, and affective components of the 35 pre-service mathematics education students who had their first-hand experience of MI through online PBL. The students were given two contextualized, ill-structured problems, worked in groups, and proposed solutions to the problem. Data from the teachers' observation notes, students' learning logs, and the transcripts of the focus group discussions were gathered. Qualitative data analysis suggests that students were involved in peer-to-peer learning that included task-related conflicts, pattern recognition, self-regulation, critical analysis, participation in the learning activity, attention to the tasks, and interaction with the learning community which led to their enjoyment and motivation in conducting MI. The extent of their engagement during the presentation and analysis of the problem, planning and developing a solution, presenting and assessing solution, and summary and debriefing were further analyzed. The findings revealed that while students were initially puzzled at the start of their investigation, they shared, welcomed new information, analyzed it, and valued different approaches to a challenging mathematical problem as they progress with their inquiry. The results also revealed respondents' perspectives on how to manage online PBL. These findings can help teachers design collaborative online activities.

Introduction

The mathematics curriculum's framework strives to foster critical thinking and analytical thinking (SEI-DOST & MATHTED, 2011). Problem-solving, communicating mathematically, reasoning, and making mathematical connections are all parts of critical and analytical thinking. The objective is to attain the focus goal by imparting sound mathematics knowledge to all Filipino students, regardless of their background or circumstances, while also fostering the growth of strong cognitive abilities and the development of positive cognitive values. Mathematics serves as a solid foundation for the concepts and life skills that Filipino students learn in basic education because it is a skills-based topic.

Students should participate in problem-solving activities where they define the objectives, come up with plans of action, show how the knowledge or concepts were used, and explain their reasoning. Students gain new knowledge through this activity by figuring out what other students are thinking and coming up with a viable solution to the problem. Through problem-solving and mathematical investigation, students gain a deeper understanding of mathematical ideas and develop mathematical thinking skills.

Through inquiry, analysis, and study, a learner undertaking a mathematical investigation investigates a complex problem that is mathematical in nature. An investigation's distinctive feature is the notion that the student is responsible for formulating and pursuing the question (Kissane, 1998). It covers the core concepts of mathematics, including how to formulate problems, cope with situations for which there is no known solution, and create and test hypotheses. There have been efforts to include investigation into the mathematics curriculum due to the potential for increasing mathematics learning.

Vygotsky's social constructivism and Garrison et al.'s (2010) community of inquiry, cited in Paz and Pereira (2015) served as the foundation for the Problem-Based Learning model and Mathematical Investigation as a problem-solving method. The social constructivism theory suggests that in a meaningful mathematics course, the students construct their knowledge. Learning occurs when the new knowledge is presented in relation to the existing knowledge within an effective learning environment and the new knowledge is practiced through experience. In addition, Garrison asserted that learning in an online setting is based on three elements of a community of inquiry: social presence, cognitive presence, and teaching presence. The manifestation of these three defines the learner's educational experience. The social presence refers to a member's ability in the community of inquiry to present their characteristics, depict their role in the community, create relationships with other members, and exchange ideas in an online classroom. The cognitive presence is the extent to which learners can construct knowledge through exchanging information, connecting ideas, and applying new ideas. Teaching presence is defined as the design and facilitation of cognitive and social processes to achieve desired outcomes (Oyarzun et al., 2017). Hence, student's learning and understanding of the mathematical investigation is influenced by their engagement and participation in the PBL tasks.

Students' engagement is multidimensional in nature, encompassing cognitive, behavioral, and affective components (Bond et al., 2020). The cognitive dimension encompasses peer-to-peer learning, pattern recognition, self-regulation, and critical analysis. The behavioral components include participation in the learning activity, confidence, study habits, and attention to the tasks or lesson. Positive interaction with the learning community, enjoyment, and motivation are all indicators of the affective aspects of engagement.

The existing literature has opposing views on students' engagement in online learning during the COVID-19 pandemic. The study of Cleofas (2021) asserted that there is a high online student engagement, however, the studies of Hollister et al. (2022) and Wester et al (2021) claimed otherwise and their low engagement negatively influence their learning experience. This study aimed of investigating the participants' lived experience as they engage in mathematical investigation (MI) through online Problem-Based Learning (PBL).

The four major steps of problem-based learning (PBL)—presenting and analyzing a new problem, planning, and designing a solution to the problem, presenting and analyzing the answer, and summarizing and debriefing—were introduced to students in an online setting. A PBL problem that acted as the main motivator for learning was presented to the students. Together, they planned and created a workable solution to the problem. The other students in the class were then encouraged to pose questions to help clarify the solutions as each group had a chance to present their approach to the problem. The purpose of this phase is to help students better comprehend mathematical concepts and the investigative process, since the activity gives them a chance to express their thoughts. After completing the activity, the class participated in a summary and debriefing to help everyone develop a shared understanding of the lesson. The exploration of the students' participation and engagement in the PBL activity was conducted.

Materials and Methods

This study, which focuses on investigating the lived experiences of the pre-service teachers that have affected their understanding of the mathematical investigation, was phenomenological in nature (Horrigan-Kelly, Millar, & Dowling, 2016). The participants' cognitive, affective, and behavioral engagements in the online PBL as they work through MI to address ill-structured problems were documented. This approach was utilized to examine the similarities between their engagement as they examined the problem, planned, and developed a solution to the PBL challenge, presented their suggested solution, responded to inquiries, and listen to other group's solutions to the same PBL problem.

It was conducted at a State University situated in the Philippines and participated by 35 Bachelor of Secondary Education major in mathematics undergraduate students enrolled in Math 315-Problem Solving, Mathematical Investigation, and Modeling in the first semester of the academic year 2020-2021 with the asynchronous online learning (AOL) approach. They were senior high school graduates and had passed the college and advanced algebra, plane and solid geometry, modern geometry, and logic and set theory courses. Also, it was their first time enrolling in Math315. During the study, the majority of them (n = 25) worked part-time, most of them (n = 27) studied at night, and they used their mobile phones (n = 33) or laptops (n = 18) for online learning with prepaid data (n = 19), home internet plans (n = 13), or shared Wi-Fi with neighbors or relatives (n = 3).

As the participants had no prior experience in MI and PBL, the initial online meeting with student respondents was intended to familiarize them with the mathematical investigation and to orient them to the study. A lecture-discussion was held to define and discuss the processes of mathematical investigation. Moreover, students were provided with a semi-structured investigation task involving Pascal's triangle to familiarize them with the process of mathematical investigation. The teacher-researcher guided the learners in identifying patterns in Pascal's triangle by asking questions. Following the activity, the teacher lectured on the definition and procedures for mathematical investigation.

After the lecture, students were informed about the purpose of the study and their role in attaining its goal. They were oriented to problem-based learning through videos and discussion on the essential aspects of PBL. The teacher-researcher emphasized the functions of the students and teacher to maximize the benefits of engaging in PBL.

Before the end of the meeting, the students were asked to form eleven groups. A group must have three members, except for two groups with four members since the class size is 35. The teacher-researcher allowed the students to select their groups. Rusticus and Justus (2019) argued that this was the better option than the teacher-selected teams when the students already knew one another. The group's list was submitted to the teacher-researcher before the next meeting.

The students participated in six weeks to solve two PBL problems. The groups had to analyze, plan, develop and propose a solution to each problem in two weeks, and present their solution in the third week. The teacher-researcher began by presenting the course goals and reminding students of the scheduled tasks for the lesson duration. Then the researcher introduced the PBL problem, and the students were given a few minutes to re-read the situation. Then, the researcher guided the students in developing their understanding of the problem through questioning. Following this, each group was directed to a teacher-assigned breakout room to meet and share ideas about the problem and plan for the course of actions they would take to develop a solution to the problem. Moreover, the teacher occasionally joined the group to guide the students in developing their thinking.

In the second week in the PBL cycle, the students developed their solution to the PBL problem. Since the learners worked asynchronously, the teacher reminded them of the tasks to accomplish and the outputs to submit at the end of the week. Presentation and assessment of the solution to the problem and a summary of the activity commenced in the third week through synchronous online learning sessions. Before the meeting, each group submitted a copy of their investigation report electronically and posted a video recording of their investigation in the Google Classroom Stream. This allowed the class to view other groups' investigations and ask questions about the proposed solution.

Students recorded their learning experience and reflections on the exercise on their learning logs. These were similar to journals that students keep in which they reflect on what and how they learn. Moreover, online student interviews were conducted to get information about their MI experience through PBL. In addition, three mathematics teacher who has prior experience teaching MI observed the synchronous online meetings and recorded their observations of the noticeable student behaviors.

The qualitative data gathered from teachers' observation notes, students' learning logs, students' interview transcripts, and video recordings from the online class sessions were analyzed to determine and describe their engagement. The data was coded and examined to identify emergent themes across the cognitive, affective and behavioral facets of student engagements. The researchers utilized the eclectic coding process in Saldana (2013). Using this method, the researcher employed a combination of descriptive, in vivo, and procedural coding methods. The codes were then reorganized into categories to identify patterns and formulate emergent themes.

Results and Discussion

The discussion of the results follows the PBL phases used to develop student's understanding of MI. It begins with the presentation and analysis of the problem, followed by planning and developing a solution, presenting and assessing solutions, and summary and debriefing.

Empowering Students through Collaborative Inquiry: Increasing Motivation and Goal Setting

The students were puzzled and initially reluctant to share their ideas, but after engaging in a collaborative discourse in which they asked questions, provided examples, and activated math concepts that might be useful in their inquiry, they became motivated to explore the problem; they set goals and timelines for completing their goals. The students set learning goals and the timeline to accomplish those goals at the beginning of the PBL activity. As the teacher reminded them about the learning outcomes expected of them, the activity timeline and their role in achieving learning, they set their target. The reminders were viewed to be essential to the students to keep on track with their learning. A student recognized the importance of reminding them, as reflected in their learning log entry: *"... I learned that objectives teach students to have a sense of accountability. Giving us the target goals for a particular task teaches us how to be responsible and accountable in accomplishing our work."*

Setting goals and communicating them to students encourages them to evaluate their assessment results and use their learning to propel their progress over time (Nordengren, 2019). By sharing learning objectives with students, they become aware of the learning that they need to acquire. In addition, students were informed about how their learning would be assessed. Their awareness prompted students to regulate their learning, reflect on their experiences as they engaged in the activity, and convert their learning into a desire to study more to accomplish the learning goal.

The students were puzzled by the problem scenario because it was the usual problem that they solve. A student wrote, *"I am having a hard time thinking about how to solve it. It is not the typical problem in math that you need to add, subtract, multiply, divide, or find the area of a shape."* It was also found out that this was their first time dealing with an open problem, and thus, they were shocked and puzzled since they thought that there were insufficient details in the scenario. On the other hand, students knew what to expect when the second problem was given. Even so, they find the problem tricky and complex, but also thrilling and enjoyable, as it incorporates art.

A problem-centered scaffolding technique (Antonenko et al., 2014) helped students define and explore the problem. The questions and instructions were given to guide students in identifying the problem goals and conditions and articulating the available and needed information. Using the guide questions and communicating ideas helped students identify the problem's assumptions and goals. After the presentation of the problem, students were given time to get a grasp of the scenario. Then, guide questions were given to assist students in identifying the problem, activating prior knowledge by identifying known conditions and collecting the necessary information about the situation. Students gained an understanding of the problem by answering the guide questions.

Students analyzed the problem trough reading the problem a number of times, asking questions, sharing ideas, and giving examples. While analyzing the problem, students raised questions to clarify ideas. They also developed common understanding of the problem through collaborative discourse. Asking questions aided the student to identify and organize the solution-relevant components and relationships stated in the problem while building a situational picture. In addition, answering questions aimed of contributing to their increased understanding of the problem condition (Rellensmann et al., 2017). These forms of engagements were found to be helpful in their analysis, as reflected in their learning log entries:

> *Rod: We had a sharing of thoughts about the problem with the class. We used menti.com and answer the questions or give our opinions of the things that our teacher asked. By this, we realized other things and it helps us think of many other ideas.*
>
> *Anne: First, it was so hard for me to understand the lesson because I don't know what we need to do. But when it shows the picture, I finally get what we need to do. I enjoyed learning how to place the strings in the pins with the specified gaps.*
>
> *Joyce:Reading the string art problem again and again gives me additional understanding about the problem. It helps me to analyze every detail of the string art problem. By making an illustration about the total number of strings needed for a certain number of pins and gaps, helps me to make a plan on how we are going to start to make our possible solution that can fit to the conditions of the problem. Sharing our illustration for the total number of strings needed for a certain number of pins and gaps, help us to analyze the possible strings and pins that can be used in the string art problem.*

Reading and responding to questions about the problem is beneficial for gaining a grasp of it. Joyce viewed the string art problem differently after reading it a few times. She further asserted that analyzing the problem provided a valuable information and that illustrating string art supported her in assessing possible string and pin solutions. By responding to questions, the students learned more about the problem while also finding enjoyment in their work utilizing the online assessment tools. The following were some of the students' insights about the activity as reflected in their learning log:

> *"With that investigative problem, we tried to generate ideas by answering the guide questions provided by our professor through the use of Mentimeter. This is my most awaited part in every meeting on this subject because I find it enjoyable and it helps me a lot in understanding the problem."*
>
> *"I enjoyed what we did in the last discussion when we draw in the jam board. It was my first time doing that and I enjoyed it a lot. Even if I'm not sure with what I am doing, I think I did a good job."*
>
> *"First, it was so hard for me to understand the lesson because I didn't know what we needed to do. But when it shows the picture, I finally get what we need to do. I enjoyed learning how we need to put the strings in the pins with the given gaps."*

The teacher's questioning using online assessment tools elicited their engagement. Additionally, by posing open questions, students must think more deeply about the problem conditions, resulting in a better grasp of the PBL problem. Students developed an understanding of the problem under the teacher's guidance, which influenced their engagement and performance in the mathematical investigation.

A breakout meeting was conducted for students to collaborate on analyzing and developing initial plans to come up with a solution to the problem. The learners admitted during the interview that they were confused and unsure about what they needed to do in the breakout area. They were at a loss for what to do because this was their first exposure to an open problem. They said that it was only when the teacher joined them and began to ask questions that they had an idea of what should be done.

By participating in discussions and presenting examples that match the situation, the students developed their understanding of the problem. They discovered some observations that could be used to kick start the problem. Students argued that the breakout session helped them gain a better understanding of the problem. Some students wrote the following about their learning log:

> *Ian: We also collaborate in generating ideas, as we share what we see and understand the problem we are seeing some observations we can used in the problem.*
>
> *Joyce: Being collaborative to your group will be much helpful in understanding the problem as we are unique individuals in way of thinking we can understand it more in more possible perspective… I've learned on the different insights and ideas of my groupmates. Their understanding about the string art problem is very essential to do our plans to arrive the best solution that can fit to the conditions of string art problem.*

The problem analysis also introduced the mathematics concepts that could be useful in the student's investigation. For instance, while analyzing the conditions of the succulent problem, Dave wrote,

> "*We have a lot of questions in our minds, but we also find it possible to respond to this situation. We've discovered that even counting, odd, even triangular numbers, prime numbers, arithmetic sequences, and geometric sequences will fulfill the first and second conditions. These examples represent a distinct number of succulent plants".*

Dave's insights about the task showed that students recognize concepts that may be the foundation of their investigation. This knowledge was activated while they analyzed the problem. The mathematical concepts that might help the students in their investigation started to come out at this stage.

Students used mathematical investigation by working with open problems. Analyzing the scenario by responding to the module's guiding questions and discussing opinions regarding the problem's goal and assumptions impacted the students' conceptual knowledge, investigation, and communication skills. This learning activity engaged the students' prior knowledge, allowing them to identify the objectives and conditions necessary to achieve them. Additionally, responding to and asking questions about PBL problem within the student groups helped them develop their analytical, reasoning, and communication abilities, all of which are important for mathematical exploration. Moreover, the selection of tools and techniques for implementing the online PBL is critical in creating a learning environment that encourages students' interest.

Building Justification and Resilience in Learning: Collaborative Inquiry to Promote Student Autonomy and Satisfaction

The students participated in a collaborative discussion with their peers or an expert; they specialized, organized facts, developed and tested conjectures, and reviewed concepts that could help justify their claims. When they were able to support their conjecture, they felt satisfied despite how challenging the work was. At this stage, students worked collaboratively to devise a strategy and develop a solution to the PBL problem, with minimal guidance from the teacher. Each group of students was tasked with meeting together to strategize on how to address the problem. Then they carried out their agreed-upon plan. The module's suggestions and questions challenged learners to examine several scenarios, explain viable solutions, make rational decisions, and reflect on their experiences.

Students specialize at the start of their investigation. They gave examples of the possible outcomes that fit into the situation. Aside from the students considering this task easy and enjoyable, they believed that specializing influenced their investigation. As mentioned by a student, *"Specializing in a problem like this is very important in order to boost the entire process of investigation until the end. It will serve as the initial foundation upon which conjectures will be built."* Another student that supports this claim said, *"Specializing helped us to further understand the problem and form a solid view of the situation. It helped us form conjecture by supposing variables."* Accordingly, specializing reinforced the student's analysis of the problem and helped them in the formulation of conjectures.

Organizing data collected through specialization assisted students in their observation. To summarize the information, some students created an organized list or drew a table. This action benefitted the students as they continue their investigation. The statements below were Rose and Ces's narration about their specializing process:

> *We gave an example, we made many examples, then make a table of pins, gaps and strings. We made a table with number of pins, gaps, and strings. Then we looked for a pattern on the table. Then, when we saw a pattern, we drew again to see if the pattern is correct.*
>
> *Creating a table with the number of gaps, pins and strings made our investigation easier. We struggled a bit at first but we eventually able to find a pattern within the table. We are able to find a possible combinations and patterns in the table that will fit the conditions of the problem.*

After organizing the data, students analyzed them and noted the evolving patterns. From their observations, students formulated their initial conjecture and followed it up by testing and justifying it. Students considered this investigation stage to be the most challenging yet rewarding. For instance, when students were asked about their experience, Ian said:

> *"Conjecturing is challenging too. This is because this is the start in the investigation process that I need to exert all the observations to be able to make a conjecture. In conjecturing, this is the part where you weren't sure enough about what you were putting. It's like it's hard to be assured that what I stated is right. It's not that I don't like that part, I'm just not confident… I'm not sure if what I put in is right… I'm not sure if the conjecture leads me to the right direction in the investigation. When I make a mistake at*

this point, it's like going back from the start of the investigation.

I liked the justification part of the investigation the most. First, because justification is challenging, we have to go through quite a lot to be able to justify. As a group we made a justification, it was really hard to come up with a simple justification, so we had to think and think and think. But not because it became complicated, so I liked justifying… I liked it because when we justified a conjecture, that was the most fulfilling part of the investigation because you can tell yourself that its proven, after the tasks that we did, after the challenges, after our hardships, that we justified it in such a way that other people will understand, and they will know that what we did was right."

Ian's view represents most of the students in the present study. They struggled and were uncertain about their conjectures, and they could hardly justify them.

Students perceived the task as a complex process since it requires a strong foundation in mathematical concepts and analytical and reasoning skills. Moreover, Ian was aware that justifying conjectures entails using correct arguments so that his classmates would understand their investigation. It was evident that the students were also developing their communication skills.

Students employed several means to help them justify their conjectures. This study identified that reviewing concepts, collaborating with their peers, teacher's mentoring, and regulating their learning were beneficial in justifying their claims. Most of the students reviewed their previous mathematics lessons, particularly on topics used in their investigation and lessons related to logic and reasoning, to gain knowledge that would strengthen their justification. They reviewed topics such as factors and multiples, divisibility, algebra, probability, geometry, and number theory by referring to their notes from previous math courses and searching online for additional resources. Additionally, they used online databases related to set theory and logic to help with their reasoning. Students learn and re-learn the mathematical concepts and logical procedures that lead to the advancement of their foundational knowledge.

Students worked collaboratively through Facebook Messenger or Google Meet to discuss the progress of their solution. Often, a learner shares their observation while the others listen and comment or ask questions about the idea presented. Joyce shared,

"*We do a brainstorming, sharing our opinions and ideas to create a better solution that can fit into the given situation. Sometimes listening to the ideas of each of my group members gives confusion because we throw some queries about the ideas of each other on how we are going to defend our suggestions in the number of strings and gaps and the other conditions given by the given situation."*

It can be deduced that collaboration is present in Joyce's group, where they engage in active dialogue by sharing, listening, and asking questions.

Students stressed the significant role that collaboration played in their learning process. After engaging in collaborative discourse, students were more prepared to organize their ideas. They explained what they know or think they know, while also making connections between the existing and newly acquired knowledge. Additionally, openness to new ideas and listening to understand a specific piece of knowledge is just as essential as sharing information. It entails paying attention to content knowledge to build new knowledge from the group's evolving ideas (Remedios et al., 2012). Students must practice speaking and listening skills with classmates in order to process what they have learned (Stanford & Henderson, 2016). Students develop their ability to think independently through collaborative conversation experiences that require them to carefully analyze arguments and determine whether they agree or misunderstand information.

Sometimes, collaborative discourse results in conflicts that lead to arguments. For instance, Marie and Dave shared the following when asked about the challenges they encountered in the investigation process:

> *Marie: We seriously talk about our conjecture. When giving an opinion, we talk. Since we are in a group, when someone has an opinion, other will also oppose or say something that may be wrong or lacking in one's idea. In other words, we are the ones who deal with our problem. We argue, have a healthy discussion. It's like as a friend and you see what's wrong with him, you're the one to tell him more than anyone else knows. So, for us it is better for to talk about the mistakes in the ideas before our presentation. We are the ones who oppose someone's argument. Then we will make a resolution.*
>
> *Dave: We usually debate, we compare our answers. Whoever is correct, his answer will be accepted, and he will explain how he got the answer...We have a healthy exchange of ideas. Friendly debate is happening between us.*

There were task-related conflicts between Dave and Marie's groups. Six groups shared during the interviews that they also had disagreements about some ideas, especially when they were writing and justifying their conjectures. Due to a conflict, the students had to listen to others' ideas, reconsider their own, collaborate, and construct them collectively. It enables group members to put their ideas to the test by subjecting them to critique. However, their disagreement related to their conjectures, or the investigation task in general, positively affects collaboration (Lee, Huh, & Reigeluth, 2015). It promotes argumentative skills and provides an opportunity for students to gain new knowledge, address misconceptions, or deepen their understanding of existing knowledge.

The online learning arrangement also affected students' behavioral, affective, and cognitive components of their engagement in the mathematics investigation through the PBL activity. Their conviction and focus on the task diminished while they asynchronously worked on formulating, testing, and justifying conjectures. During the interview, students from three groups said they had a hard time communicating with their peers because scheduling a time for them to work at the same time was challenging due to other responsibilities, both in school and at home, that they had to fulfill. Hence, on some occasions, a student focusing on the inquiry wanted to provide information about their investigation and seek validation of the idea but could not receive prompt feedback. That resulted in a delay in completing the task and less motivation to advance with the investigation, while wishing that the learning set-up would be back to face-to-face. While some other students had similar challenges, they tried to explore other ways to receive feedback or deal with their problems.

During the asynchronous learning, the teacher established a plan for motivating and monitoring students' progress. Reminders were posted in the Google Classroom or Facebook Messenger. Students were reminded about the tasks to be performed and the outcomes to be produced during the week. A student understood the value of being reminded to manage their time well to complete the assigned tasks. She mentioned, *"Reminders are relevant for us, students. As our professor told us the reminders yesterday, I realized that I need to be mindful of several things, like time, which I am lacking during these times."*

Time and self-directed learning skills are essential for students to adapt successfully to online learning (Baum & McPherson, 2019). Unfortunately, the study's participants admitted to lacking these skills. The learners and their families were still adjusting to the online setup. For instance, a student shared, "*Because at present, ma'am, while I was making a conjecture in the house, I will be asked to do other household chores. We cannot focus. The only problem we had was the situation, so we ran out of time*." To assist students in overcoming this challenge, the teacher reminded them of the expected outputs at the start and throughout their investigation.

While developing the solution, most student groups seek confirmation of their conjectures from knowledgeable others. The majority of questions asked through messenger were to ascertain whether their conjecture was correct or to inquire about the submission timeline. Additional questions that require discussion were raised during online consultations. Six groups requested mentoring during the first investigation activity, while nine groups requested a consultation meeting during the second teaching experiment. Each meeting ranges from 15 to 30 minutes. On the other hand, during the interview, students revealed that their lack of confidence in asking and discussing their work and time constraints due to their work were the reasons for not consulting the teacher. Through the teachers' questioning about their evolving ideas and conjectures, students discover how to improve their conjectures or explain their work. By exploration of other sets of numbers, the group came up with a better analysis of the problem being investigated.

Students collaborated to develop a solution to the PBL problem. In this PBL stage, students engage most in a mathematical investigation. Students were required to collaborate in an online format, along with the difficulties inherent in the investigation process. Also, being stuck during the investigation is unavoidable. At a certain point in the investigation, students think that they are not making any progress or that their strategy is not working (Mason, Burton, & Stacey, 2010). Reviewing previous lessons in concepts used in the investigation, collaboration, and coaching from the teacher gave students the chance to engage in the learning tasks by recalling concepts, addressing misconceptions, resolving conflicts, thinking critically, finding patterns that led to the development of conjectures, regulating their learning, and asking help from their peers and their teacher. Through collaborative discourse, students shared and defended their observations and listened to and challenged their peers' ideas. Engaging students in this inquiry-based setting has several advantages, including improved content learning, development in reasoning, and higher motivation (Duncan, Av-Shalom, & Chinn, 2021), as well as critical thinking skills (Wale & Bishaw, 2020)

Developing a Growth Mindset: Promoting Self-Reflection, Openness to Multiple Perspectives, and Confidence in Student Inquiry

The students were proud of their work while also acknowledging that there were different conjectures and answers to the problem. They enjoyed writing their reports, saw opportunities for improvement, and anticipated questioning about their solutions. Students submitted a short video presentation and a written report summarizing their investigation. Also, students were instructed to post the videos before the class meeting, allowing other students to view their solutions to the problem. Then, the online learning session focused on presenting the important aspects of each group's investigation and questioning their report.

Students enjoyed preparing the report since they felt they were nearing completion of the complex tasks while also learning. As the students prepared their reports, they identified other ways to enhance their conjecture's statements or arguments to support their reasoning. For instance, Dave reported,

> "*We seem to be OK with our written report, and when we were recording the presentation, we were able to come up with a better idea on how to state the conjecture or the justification." These prompt improvements in the written report and video recordings. We had a lot of fulfillment; it was very fun because the output of our investigation was good.*"

Aside from organizing the report, students also prepared for questioning. Several students disclosed that their groups met a few hours before the synchronous class meeting. They anticipated possible questions that might arise from their investigation and prepared answers to those questions. These actions developed the students' analysis, reasoning, and communication skills.

Mixed emotions were felt during the presentation and assessment of the group's solution. Rose, for example, wrote,

> "*It feels great to be able to face our fears as it helps us overcome them by listening to others' suggestions and corrections. When we conducted our investigation, we discovered new things and ideas that we could use in the future*."

Rose believed that they gained new knowledge from the activity, but they also admitted that they were anxious. They were afraid that they would not be able to answer questions, and they were also worried that their solution would not be acceptable.

The teacher's facilitation of the activity is an essential element in addressing the learners' anxiety. For instance, Rain added,

> *Even if they say don't be nervous, we are nervous. What hurts us is that when our conjecture is wrong, what we work for is wrong... But we like the approach: even if there is something wrong in our work, it is not totally rejected; we were told that we only need to modify something. That approach is good, even in a video presentation. Even if we made a mistake in GCM, the approach to us is good."*

Rain's perspective implies that the teacher was able to foster a healthy classroom culture in which making mistakes is seen positively. Presenting solutions is not easy, but it offers an opportunity for correcting mistakes and learning. Moreover, Wang and Zhang (2020) asserted that feedback coming from the teacher had a greater chance of increasing students' participation and performance.

Listening to other groups' solutions is surprising for the students. They were amazed by the other possible ways of dealing with the problem and the other conjectures that other groups developed. Students understood the concept of mathematical investigation and other possible ways of solving an open problem. They were informed of other ways of looking at the problem, formulating conjectures and justifying them. Some of the students' perspectives on their learning logs were as follows:

> *I saw different strategies to form their conjectures. And I admit that I learned from their different solution. Imagine in one problem there have a lot of possible solution that are made by each group.*
>
> *I also learned in my classmate video presentation because we have a different kind of conjectures and the presentation, they did is very nice and clear. In our class we create a lot of conjectures in one problem, and I just can't imagine we did it. We also used a different kind of formula so we can solve the conjectures and problems we do.*
>
> *I also learned the different ways of solving the problem in the findings of my classmates. It gives me an impression of, wow it is also working!*
>
> *I learned that there are so many conjectures that can be applied and there are many ways to justify them. It becomes clearer to me that each group have different perspective in this problem.*
>
> *We all have almost different conjectures and solutions, and from those conjectures and solutions, indeed I realized and learned that "Ah-ha, those could also possibly solve the investigation!"*
>
> *I've learned a lot especially in the works of each group. Imagine in one problem there have a lot of possible solutions that are made of each group.*

Students demonstrated a favorable attitude towards the presentation and assessment of their investigation. They were astonished by the diverse approaches to investigating the problem, formulating, and justifying conjectures. Through this learning activity, students learned the math concepts applicable to the problem and identified opportunities to enhance their competence in MI.

Developing Knowledge and Growth in Mathematical Investigation: Leveraging the Power of Reflection and Perseverance

The students focused on the activity, reflected on their experience, and were determined to improve their skills for carrying out a mathematical investigation. Following the presentation of each group's solution to the class, the teacher summarized the activity and the students' proposed solutions to the problem. The student's conjectures were summarized and analyzed, which resulted in identifying similar or related conjectures. The teacher mentioned best practices from some groups that the other students may follow. Aside from the strengths, the teacher also identified the student's weaknesses concerning their investigation.

This presentation did not only add knowledge but also motivated students to develop their proving skills. One student said, "*... after our teacher showed her proofs and conjectures, I became envious and amazed as well. I really wish that I could investigate like the way she did*." The student clarified that the enviousness that they felt *"will become a fuel in the succeeding mathematical investigation"* and will challenge them to strive to learn more about and develop their reasoning skills.

The students reflected on their mathematical investigation experience. They thought about their learning and identified their strengths and weaknesses to improve their performance for future investigation. For instance, Rain wrote in his learning log,

> "*I learned that the conjectures must be made simpler because it will be easier to understand. I learned this because I noticed that almost all groups present their conjecture in a complex manner. I also noticed that the teacher commended those groups who made their conjectures simpler but direct to the point. Furthermore, after my teacher showed us the summary of the conjectures that we made and a possible way of proving a conjecture, I realized that there were many things that were lacking in our investigation, and I am hoping to investigate a problem like my teacher did*"..

This reflection suggests that the assessment of the group's investigation motivated the students to think about their work and improve their performance in their future investigations.

Conclusions and Implications

The PBL activity engaged the students in solving an open problem through mathematical investigation. Despite various challenges such as unfamiliarity with dealing with open problems, anxiety, and low confidence in the online discussion, managing their time, especially while working synchronously, and regulating their learning, they were able to cope with the online learning hurdles. The utilization of online assessment tools escalated their motivation. Taking initiatives, peer support, the teacher's support throughout the learning activities both in synchronous and asynchronous ones, and positive interaction with the learning community fostered enjoyment, motivation, and critical analysis.

By informing students of the learning objectives, they were able to establish their own goals, plan their learning processes, and engage in self-learning, all of which are traits of self-regulated learners (Zimmerman & Schunk, 2011). They established goals for themselves for the day or week. Further, setting goals motivated them to plan, organize, and complete tasks. They structured their learning according to their preferences—whether they studied alone or collaboratively or worked synchronously or asynchronously. They chose resources that would assist them in undertaking their investigation, especially in justifying their conjecture. Through reflection, they assessed their strengths and limitations in studying. In addition, they chose mechanisms to compensate for their learning deficiencies. They conducted research or asked for help from peers or a mentor. They were able to manage their time effectively and efficiently by regulating their learning. These self-regulated strategies were associated with the students' mathematics performance (Harding et al., 2019; Fauzi & Widjajanti, 2018).

Through collaboration, students applied their analytical, reasoning, and communication abilities to improve their knowledge. Sharing, listening, and questioning thoughts among the students enables them to identify the problem, establish plans, elicit information, select the most appropriate solution to the problem, and evaluate their investigation process. Dialogue within collaborative groups allowed students to explain and justify their plans and ideas and understand others' views. The exchange of ideas is essential for developing the process skills needed in a mathematical investigation.

Based on the result of the study, the teacher has a critical role in designing activities that will promote students' engagement which has a significant impact on the development of mathematics learning. They should clearly inform the students of the learning goals, the teacher and students' role in their learning. Further, the teacher's role as a knowledgeable other, as motivator, and a critic is a key element in encouraging students to be engaged in the investigation activity.

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Author Information

Ellenita Garcia Manalaysay

https://orcid.org/0000-0003-2486-5757

Bulacan State University

College of Science

Mathematics Department

City of Malolos, Bulacan

Philippines

Contact e-mail: *ellenita.manalaysay@bulsu.edu.ph*