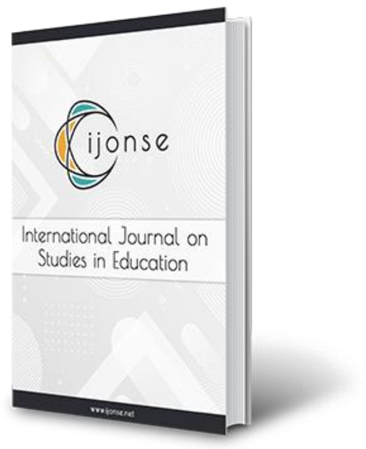

www.ijonse.net

Concepts Most Successfully Learned and Most Challenging to Students in a Redesigned Pre-Calculus Course

Angel Perez
California State University, Dominguez Hills, United States of America

Ha Nguyen
California State University, Dominguez Hills, United States of America

To cite this article:

Perez, A. & Nguyen, H. (2024). Concepts most successfully learned and most challenging to students in a redesigned pre-calculus course. *International Journal on Studies in Education (IJonSE), 6*(1), 177-190. https://doi.org/10.46328/ijonse.194

International Journal on Studies in Education

2024, Vol. 6, No. 1, 177-190 https://doi.org/10.46328/ijonse.194

Concepts Most Successfully Learned and Most Challenging to Students in a Redesigned Pre-Calculus Course

Angel Perez, Ha Nguyen

Article Info

Article History

Received:
29 October 2023
Accepted:
29 December 2023

Keywords

Pre-calculus
Active learning
Gatekeeper
Redesigned pre-calculus

Abstract

Pre-Calculus is considered one of many gate-keeper courses for STEM college students (Viera Jr et al., 2019). To better support students in Pre-Calculus, various college-level institutions have redesigned their Pre-Calculus courses to address learning challenges and inequities faced by students. For instance, Jones & Lanaghan (2021) redesigned the Pre-Calculus course to incorporate a standards-based grading system, active learning practices among students, and emphasis on developing a growth mindset. Students in their redesigned Pre-Calculus course passed the class at a higher rate than those enrolled in the traditional Pre-Calculus course, with a difference of 38.3% higher. Additionally, students in the redesigned Pre-Calculus course passed their Calculus I course at a higher rate, with a difference of 3.1% higher. Despite the overall improvement and encouraging outcomes to the course, further learning challenges still need to be addressed. This study seeks to answer the following research questions: *1) What concepts are most successfully learned by students in a redesigned Pre-Calculus course? 2) What concepts are most challenging to students in a redesigned Pre-Calculus course?* Data from 59 students enrolled in all sections of a redesigned Pre-Calculus course in Spring 2023 were analyzed quantitatively, using descriptive statistics. Discussion, limitations, and future research for teaching will also be shared.

Introduction

Pre-Calculus is considered a STEM gate-keeper course for STEM college students (Viera Jr et al., 2019). Students express a variety of reasons for dropping the class, such as dissatisfaction with the lecture-based teaching style of the class and the lack of a positive learning environment in the classroom (Jones & Lanaghan, 2021). Additionally, due to the COVID-19 pandemic, students were forced to engage in self-regulated learning without proper access to academic resources (George & Gallagher, 2022). As a result, students became dissuaded to further pursue a STEM degree. Colleges and universities are producing unqualified STEM scholars in the work field, particularly women and minorities, which can have a negative impact on the U.S economy (Viera Jr et al., 2019). Efforts have been made to increase the number of students mastering Pre-Calculus concepts and ready for Calculus courses. For instance, Jones and Lanaghan (2020) redesigned their Pre-Calculus course to include a standards-based grading system, active learning, and an emphasis on developing a growth mindset.

Literature Review

Student Learning in Traditional Pre-Calculus

Mkhatshwa (2021) describes *traditional learning* as the common practice of listening to a lecture and taking notes, and then allocating time to study the covered material at home. Studying the covered material at home is mostly achieved through completing homework assignments. However, without effective studying techniques, students can have a low understanding of the material. In fact, students can set themselves up for failure in future Calculus classes. Nasir et al. (2017) conducted a survey to the lecturers at the Universiti Teknologi MARA (UiTM) Perak, Tapah Campus in Malaysia to receive insights on the student teaching and learning of Pre-Calculus. Overall, lecturers have expressed that only 30% of their students have a good performance in this subject. The lecturers in the study shared that 63.6% of students tended to memorize concepts rather than understand them. More specifically, students memorize formulas because they only aim to pass their exams. As a result, Nasir et al. share that students are missing key information that can help them determine the correct method to use to solve a problem. This is especially true when students are tackling problems in Calculus I, which often involve a mixture of methods. Additionally, Nasir et al. shares that 54.5% of students come into the course thinking that Pre-Calculus is hard and that there is no point in trying. Therefore, students lack creativity and imagination skills when visualizing Pre-Calculus concepts, thus struggling in their traditional Pre-Calculus class.

As students continue to pass through Calculus with fears and deteriorated motivation, they become bored of the class and even question if the class will help them succeed in life. Thus, the instructors are advocating to place low-performing students in remedial math classes. However, that may be an obstacle for students under the California State University (CSU) system (Goyer et al., 2020). The CSU chancellor passed an executive order to remove all remedial math classes as of Fall 2018 and place students in appropriate General Education (GE) quantitative/mathematics-related courses. Following this, the Mathematics/Statistics Department at the California State University, Monterey Bay (CSUMB) redesigned all courses that fall under this category, including Pre-Calculus. The Mathematics Department at the California State University, Dominguez Hills (CSUDH) also redesigned their Pre-Calculus course. Further discussion is shared in the following subsection.

Student Learning in Redesigned Pre-Calculus Courses

Goyer et al. (2020) redesigned the Pre-Calculus course at CSUMB to emphasize reading apprenticeship and complex instruction. *Reading apprenticeship* is an instructional framework that helps students develop “discipline-specific reading and problem-solving skills needed to become successful readers” (Goyer et al., 2020). It supports students in all aspects of communication: reading, writing, speaking, and listening. For Goyer et al., reading apprenticeship in mathematics means “discussing and reflecting on emerging understandings of mathematical concepts and texts, all while developing confidence as a reader and creating a classroom space that supports these conversations” (Goyer et al., 2020). Goyer et al. implemented reading apprenticeship through in-class and out-of-classroom activities, reading assignments, and in their redesigned course textbook.

More of the general framework for reading apprenticeship can be found in Figure 1:

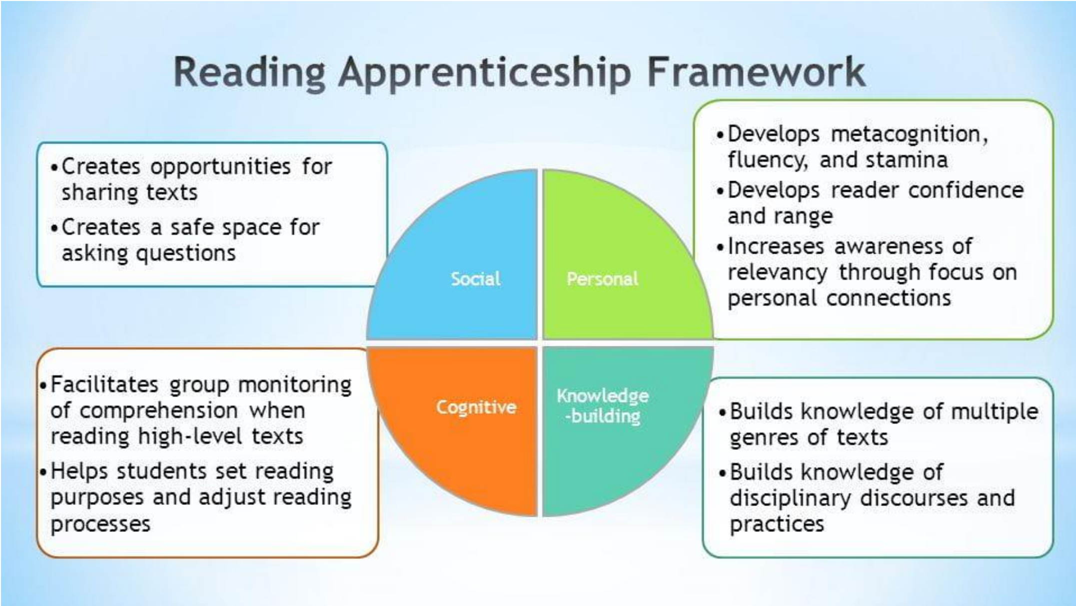

Figure 1. The Reading Apprenticeship Framework (Friendly, 2019)

Complex instruction is another instructional framework that addresses status inequities in the classroom (Goyer et al., 2020). In other words, it "enables students to learn and teachers to teach at a high intellectual level in academically, linguistically, racially, ethnically, and socially heterogeneous classrooms" (Stanford University, n.d.). During in-class activities, students are assigned roles in open-ended tasks to recognize intellectual contributions, which is how complex instruction was implemented. Both reading apprenticeship and complex instruction improved the quality of the course and paved the way for outstanding student outcomes.

The redesigned Pre-Calculus course at CSUMB improved rates of students passing the course, especially those of low-performing students. As a result of the redesigned Pre-Calculus course at CSUMB, 78.5% of students earned a passing grade, which is an increase from the 98 students in Pre-Calculus that would have taken remedial courses. Furthermore, students deemed "unprepared" passed at a higher rate than the historical average for students who had taken remedial courses before Pre-Calculus. This demonstrates the necessity to redesign Pre-Calculus to address students' learning needs and foster important skills needed to succeed in future mathematics courses and beyond.

Another study of students in a redesigned Pre-Calculus class is that of California State University, Dominguez Hills (CSUDH), which is the primary paper of focus for this presented study. Jones and Lanaghan (2021) implemented a standards-based grading system into the course, in which the instructors set the learning goals to the students, provided multiple opportunities for students to master these goals, and assigned a grade based on the goals met. Additionally, practices of active learning were implemented to the course, which included group work, classroom polling, and peer instruction. And finally, the course placed an emphasis on developing a growth mindset, the belief that one becomes smarter through effort.

During the semesters of Spring 2015, Fall 2015, and Spring 2016, Jones and Lanaghan taught seven sections of the redesigned Pre-Calculus course to 234 students, whereas other instructors taught eight sections of the course to 261 students under the traditional model. The redesigned Pre-Calculus course witnessed 88.5% of students earning a C or better. During the same period, 50.2% of students in the non-redesigned Pre-Calculus courses earned a C or better (Jones & Lanaghan, 2021). Additionally, of the 90 students who successfully completed a redesigned Pre-Calculus course, and then enrolled into Calculus I the following semester, 53 of them (58.9%) successfully passed Calculus I with a C or better. This is in comparison to the 62 students who successfully completed a non-redesigned Pre-Calculus course, with only 34 students (55.8%) passing Calculus 1 during the same time period (Jones & Lanaghan, 2021). The benefits of redesigning Pre-Calculus are astronomical, but further redesign is needed in response to the ongoing challenges faced by instructors and students.

The Pre-Calculus lecturers at CSUMB shared suggestions to improve attendance rates in the course. Lecturers in the redesigned Pre-Calculus courses at CSUMB faced challenges with attendance rates. Formally, the Pre-Calculus course was taught for two days out of the week. Now, the redesigned Pre-Calculus course is being taught for all five days of the week. Because the class is now a five-week format, students would prefer to skip class because classes would typically cover less content than in the original two-week format. Goyer et al. (2020) feels inclined to make attendance mandatory, but further challenges still need to be addressed. Thus, this study aims to 1) further investigate the impacts on student learning in redesigned Pre-Calculus courses, and 2) develop ways to better support student learning in Pre-Calculus and ultimately increase the number of students pursuing STEM degrees.

Difficult Levels of Topics in Traditional and Redesigned Pre-Calculus Courses

Nasir et al. (2017) shared that the three most challenging Pre-Calculus concepts for students, based on the lecturers' insights, are logarithmic and exponential functions, trigonometric equations and trigonometric identities, and finding the domain and range of a function. The literature review for this section shares potential challenges students face in each concept.

Frketic et al. (2019) conducted a study aimed to investigate students' learning and understanding of problems involving logarithmic functions. The study followed a mixed-methods model in order to collect both quantitative and qualitative data. Frketic et al. invited students from the Pre-Calculus course at the University of Delaware to take a pre-test on logarithmic functions. Then, 22 students were randomly selected to participate in an interview, and only seven students accepted. Finally, the students who partook in the pre-test were invited to take a post-test. The study concluded that students need to understand logarithmic terminology and include it in their explanations of logarithmic functions. The importance of understanding logarithmic terminology can be emphasized during lectures. However, Frketic et al. suggests that further research that follows a mixed-methods model on a larger sample of students is needed to develop a more effective Pre-Calculus course.

Cho et al. (2014) conducted a study that focused on students' learning and understanding of finding the domain and range of a function. The study followed a mixed-methods model to collect data similar to the study of Frketic et al. (2019). Students who participated in the study were middle-achieving mathematics students from a community college, with test scores ranging from 51% to 79%. While the participants were in the transition to mastering the concept, their insights can share the challenges that low-achieving students are still facing. Figure 2 highlights the common strategies that students use to solve problems involving the domain and range of a function.

1. Projecting the graph onto the x-axis (or y-axis) to determine the domain (or range)
2. Pushing the graph to the x-axis (or y-axis) to determine the domain (or range)
3. Focusing on the endpoints when tracing the graph from the starting point to the ending point
4. Not overlapping sections of a graph to determine its range
5. Using the closest axis value; using x-coordinate values instead of y-coordinate values, and vice versa
6. Measuring the range from the lowest value to the highest values of a piecewise function

Figure 2. Common Strategies for Finding the Domain and Range of a Function (Cho et al., 2014)

The study shared two common challenges that low-achieving students face about the concept. The first challenge is overcoming the misconception that a horizontal line or segment of a line has no range. "Their belief stemmed from the conviction that the range should have some length or distance" (Cho et al., 2014). However, the range of a horizontal line $f(x) = b$ will always be one single value: b. For example, the range of $f(x) = 2$ will be 2. The second challenge is finding the domain and range of a function with marked open or closed points as boundaries. Students find it particularly difficult to find the range of a horizontal segment that has two open points on the ends of the graph. They fail to realize that a horizontal segment has infinitely-many closed points (Cho et al., 2014). Students also find it difficult notating the range of horizontal lines. Overall, students struggle with finding the range of a function more than the domain of a function; specifically, horizontal lines. Thus, a redesigned Pre-Calculus course should involve more practice on finding the range of horizontal lines.

The last study of focus comes from Aminudin et al. (2019), in which the study aimed to determine the challenges that students face when solving trigonometry problems (see Figure 3). Twenty-nine first-year students in the mathematics education program at the Universitas Islam Sultan Agung (UNISSULA) participated in this study. Test scores were collected, and interviews were conducted among the students. Prior to the study, Aminudin et al. (2019) learned and shared the common mistakes students make when solving trigonometry problems:

1. The use of improper equations and sequences of operations
2. The misuse of sinuses and cosines
3. Focusing on the endpoints when tracing the graph from the starting point to the ending point
4. Misinterpretation of languages
5. Illogical inferences
6. Technical error calculations

Figure 3. Common Mistakes Made by Students in Answering Trigonometric Questions (Aminudin et al., 2019)

Students in the study demonstrated a low understanding of what critical thinking looks like in trigonometry.

Interviews conducted by Aminudin et al. share the common conceptions expressed by the students about critical thinking in math: “logical thinking, systematic thinking, thinking complex, arguing until new conclusions are found, formulating, understanding, and analyzing problems, and using the exact mathematical formula” (2019). Critical thinking entails many things, but because students do not follow a coherent understanding of critical thinking means that they do not fully understand what critical thinking is. Thus, curriculums for Pre-Calculus must include critical thinking practices so that students can improve their understanding of critical thinking in mathematics overall. One such practice should involve pushing students to be skeptical of the problems they are solving.

To first approach a trigonometry problem, students must be skeptical about it in order to spark motivation in solving it. According to Aminudin et al., skeptical students “identify hidden information clearly and accurately” (2019). Most of the problems that students answer in tests include false information. Students are challenged to prove or disprove the validity of the information provided through existing concepts and theorems. But while students struggle to evaluate the information provided in these problems, the lack of skepticism students provide in solving trigonometry problems is not addressed in Pre-Calculus courses. Traditional learning in Pre-Calculus typically follows the following format: Students first learn about the concept through lectures and discussions, and then they solve problems with the instructor. The practice problems completed in class emphasize how to apply formulas and use them quickly and efficiently. By the end of the class, students are assigned additional practice problems to do at home. The format provides no space for students to think critically about the problems they are completing. To achieve this, Pre-Calculus courses must address students' motivation for completing trigonometry problems.

Overall, students’ motivation can be described intrinsically in Pre-Calculus. The responses shared by Aminudin et al. (2019) have one common theme: achieving the right result and overcoming challenges. Aminudin et al. suggests using extrinsic motivation—motivation empowered by tangible and intangible benefits—to uplift intrinsic motivation. In this way, students can boost their self-actualization and sense of accomplishment in Pre-Calculus.

Purpose of the Study

This study aims to add to the research base on concepts learned in a redesigned Pre-Calculus course. The research seeks to answer the following questions:

1) *What concepts are most successfully learned by students in a redesigned Pre-Calculus course?*
2) *What concepts are most challenging to students in a redesigned Pre-Calculus course?*

Methodology

Study Site and Participants

The study took place at a highly diverse, metropolitan institution with approximately over 15,500 students. The undergraduate population includes approximately 80% under-represented groups, 47% first generation, and 58%

Federal Pell Grant eligible.

Participants in this study are 59 enrolled students in two sections of a redesigned Pre-Calculus course in the Spring 2023 semester. These were all the Pre-Calculus sections offered in the semester. The Pre-Calculus course is four semester units and meets three times per week, 75 minutes each meeting. Both sections of this course use a *standards-based grading system*, where students have multiple chances to master their 34 learning goals (22 Essential goals and 12 Proficient goals) on 10 weekly quizzes, four exams, four (optional) presentations, and one final exam. A goal is met if the student successfully solves a type of problem twice on those assessments. Homework and other types of assignments (e.g., pre-class videos) count toward their *assignment points*. Grades are assigned based on both the learning goals met and the assignment points earned. For example, to earn a grade of C, a student must 1) meet all 22 Essential goals, and 2) earn at least 70% of assignment points.

Data Collection

Data include two sets of the instructor (second author)'s goal trackers in both sections of the Pre-Calculus course. These goal trackers recorded student learning goals on quizzes, exams, and presentations throughout the semester. They contained no student information; i.e., student names were replaced by numbers. The Institutional Review Boards (IRB) at the authors' institution determined that IRB review and approval is not required for the study.

Data Analysis

To address the first research question (concepts learned most successfully), the authors found goals that were mastered the most and least by students. This was accomplished by counting the number of students that mastered each goal (i.e., answering two problems correctly of the same goal) by the end of the semester.

To address the second research question (concepts that are most challenging to students), the authors found goals that took students the longest and shortest time to master. To achieve this, the authors first reorganized the Goal Information Sheet, which was given to students (see Appendix), in a chronological order of when the assessment was given. This timeline spreadsheet (see Figure 4) outlines the goals that the students are assessed on in each assessment and the number of questions that the assessment would include for that goal. For example, Quiz 1 was the first assessment of the semester, and the Final Exam was the last assessment. A snapshot of the reorganized timeline spreadsheet can be seen in Figure 4.

The *first opportunity* students could master a goal is counted when the first two times the goal appeared on quizzes/exams. The *last opportunity* students could master a goal is counted when the last two times the goal appeared on quizzes/exams. For example, the first opportunity for students to master Goal 2 was on Quiz 1 and Quiz 3. The last opportunity for students to master Goal 2 is on Exam 2 and the Final Exam. The authors then counted the number of students that mastered each goal on the first opportunity and last opportunity. There are certain cases in which goals were assessed with three or more questions in the first/last opportunity. For instance, Goal 1 was assessed in Quiz 1 with one question, and then it was assessed in Exam 1 with two questions. In this case, Quiz 1 and Exam 1 are counted as the first opportunity for students to master Goal 1, whether students mastered Goal 1 by answering each question of the goal on Quiz 1 and Exam 1 correctly, or by answering both questions of the goal on Exam 1 correctly.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	29	30	31	32	33	34	35	36
Quiz 1	1	1	1							2													1											
Quiz 2				2	1	1								1										1										
Quiz 3		1				1		1		1				1											1		1							
Exam 1	2	1	1	2	1	2		1	1		2	2		2			1	2					1	1	1		3							
Quiz 4							1	1	1				1																					
Quiz 5													1					1								1								
Quiz 6															1	1	1												1					
Exam 2	2	1	1	1	1	1	1	1	1	1	1	1	2	1	2	1	2	1			1		1	1	1		1	1	1			1		
Quiz 7																					1	1										1		
Quiz 8																					1	1								1				
Quiz 9																			2	2														
Exam 3	1		1	1	1	2	2	2	1	1	1	2	2	2	1	1	1	1	3	1	1	1		1	1	1	1	1	1	1	1	1	1	
Quiz 10																			1			1											1	
Exam 4							1	1	1	1	1	1	2	2	1	1	2	1	4	2	1	1		1		1	1	1	1		2		2	2
Final Exam	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1					1		1	1	1	1	2

Figure 4. The Reorganized Timeline Spreadsheet

Findings

The following subsections will describe the most successful and challenging goals, as well as the goals that took the longest/shortest time to master.

Goals That Were Mastered the Most and the Least

Figure 5 shares the counts of students who mastered each goal by the end of the semester.

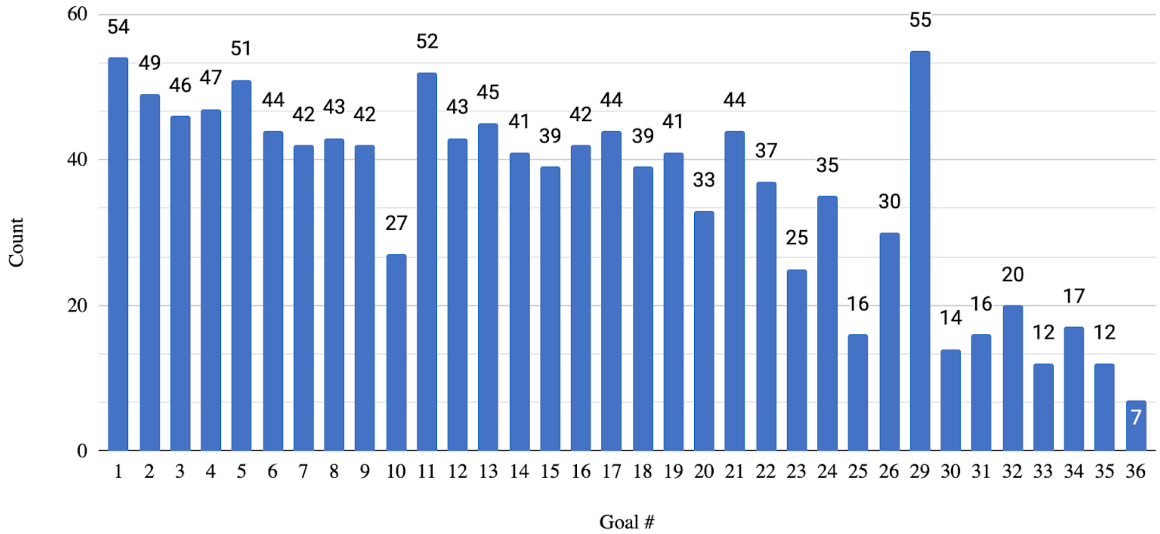

Figure 5. The Goals and the Count of Students Who Mastered Them

Table 1 shares the goals that obtained the highest counts of students who mastered it, and Table 2 shares the goals that obtained the lowest count.

Table 1. The Goals that Obtained the Highest Count of Students Mastering It

Goal	Description	Count (*n* = 59) (%)
29	Determine an appropriate function class to model a situation	55 (93.2%)
1	Solve a linear equation or a system of linear equations	54 (91.5%)
11	Determine a composition of function given in any form	52 (88.1%)

The goals that obtained the highest count of students mastering it were Goal 29, Goal 1, and Goal 11. Out of the 59 students enrolled into the course, 55 students (93.2%) demonstrated mastery in determining an appropriate function class to model a situation. 54 students (91.5%) also demonstrated mastery in solving linear equations or a system of linear equations. Finally, 52 students (88.1%) demonstrated mastery in determining a composition of function given in any form.

Table 2. The Goals that Obtained the Lowest Count of Students Mastering It

Goal	Description	Count (*n* = 59) (%)
36	Solve trigonometric equations by transforming the equation using trigonometric identities	7 (11.9%)
35	Prove trigonometric identities	12 (20.3%)
33	Solve problems involving the law of sines or laws of cosines	12 (20.3%)

The goals that obtained the lowest count of students mastering it were Goal 36, Goal 35, and Goal 33. Students overall struggled in solving trigonometric equations by transformations using trigonometric identities, with the highest-lowest count of seven students mastering the goal by the end of the semester. Additionally, students also struggled in proving trigonometric identities and solving problems involving the law of sines or law of cosines, with both goals obtaining counts of 12 students mastering them. This finding corroborates with the study of Nasir et. Al. (2017), with trigonometry being the most challenging concept that the students in the insights struggled with the most. Lastly, students struggled in solving equations involving rational functions, with the count of 12 students.

Goals that Took Longest/Shortest to Master

Figure 6 shares the count of students who mastered each goal by the first opportunity, and Figure 7 shares the count of students who mastered each goal by the last opportunity.

The majority of students were able to master Goal 1 and Goal 29 (described in Table 1) on the first opportunity, with both goals carrying counts of 50 students and 51 students, respectively. Both goals also obtained the highest count of students who mastered them by the end of the semester. Additionally, 40 students mastered Goal 5 on the first opportunity, which assesses students' ability to "model motion of objects falling with the force of gravity with appropriate quadratic functions". So far, however, we can solely conclude that Goal 1 and Goal 29 are two goals that took the shortest time to master.

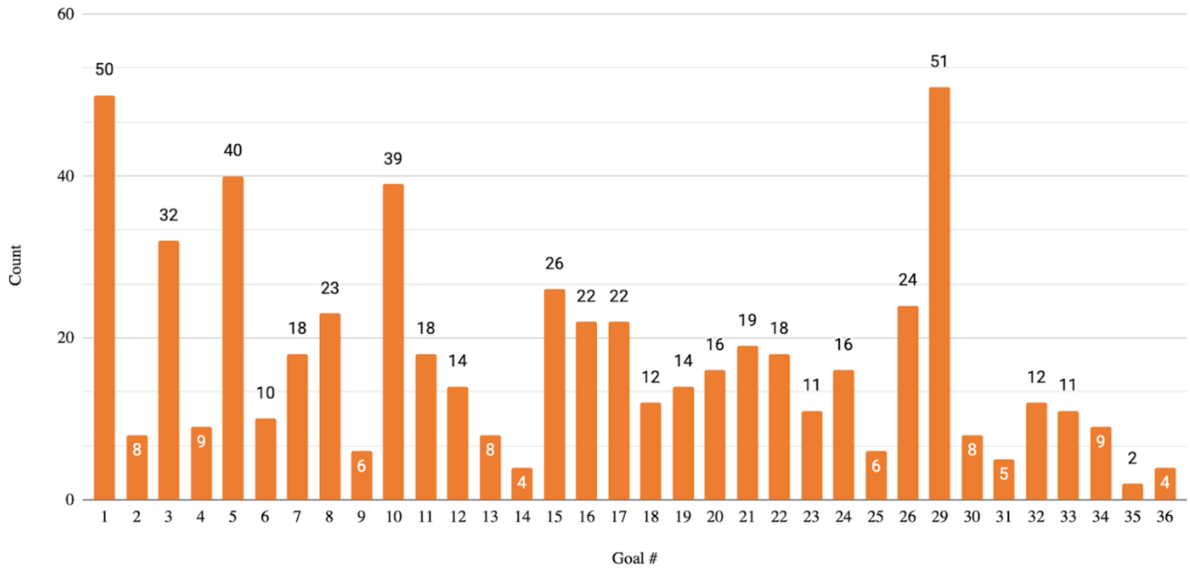

Figure 6. The Count of Students Who Mastered Each Goal in the First Opportunity

Students in the Pre-Calculus course struggled to master Goal 35 (described in Table 2), Goal 36 (described in Table 2), and Goal 14 ("Be able to give the solution to an inequality or set of inequalities using proper mathematical notation") on the first opportunity. Goal 35 received a count of two students, and Goal 36 received a count of four students. Both goals also obtained the lowest counts of students mastering them by the end of the semester. Finally, Goal 14 obtained a count of 4 students mastering it as well. The goal assessed being able to give the solution to an inequality or set of inequalities using proper mathematical notation.

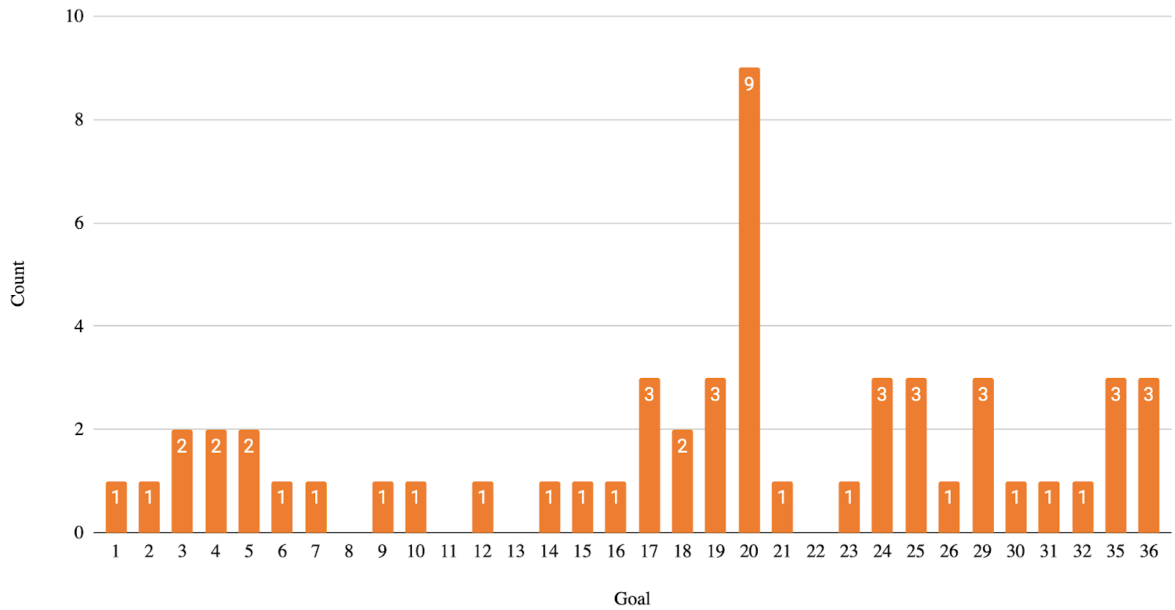

Figure 7. The Count of Students Who Mastered Each Goal in the Last Opportunity

Our goals of concern are Goal 14, Goal 35 and Goal 36, given that they all obtained the lowest count of students mastering it on the first opportunity. According to Figure 7, only one student mastered Goal 14 after the last opportunity. However, referring to Figure 5, 41 students mastered Goal 14 by the end of the semester, which is 69.5% of the class. This implies that students mastered the goal before being assigned the last opportunity. Additionally, only 12 students mastered Goal 35 by the last opportunity, but 12 students mastered the goal by the end of the semester. Goal 36 also had a low count of students mastering the goal by the last opportunity and the end of the semester, with the respective counts of three and seven students. Therefore, the authors conclude that Goal 35 and Goal 36 are the goals that take the longest time to master for students in the redesigned Pre-Calculus course.

Discussion, Limitations, and Future Research

The researchers discuss patterns found in data, limitations, and future research below.

Discussion

To determine the successful goals and the challenging goals of the Pre-Calculus course, certain factors need to be accounted for in our discoveries. The data must consider the time in which each goal was introduced. Goals 29, 1, and 11 were assessed before Exam 1, which allowed students to master the goals in preceding quizzes and/or presentations up until the Final Exam. Comparatively, Goals 33, 35, and 36 were introduced after Exam 2, which presented them with lesser time to master these goals. Another factor to consider is the number of opportunities each goal has for the student to master it. For instance, because Goal 1 was introduced in the beginning of the semester, it contained more opportunities for students to master it in comparison to Goal 36. Goal 36, however, was introduced after Goal 1 due to the structure of the course, but perhaps the goal should be introduced earlier in the course.

The COVID-19 pandemic also brought forth learning challenges for students in college courses post-quarantine (Lampropoulos & Admiraal, 2023; Stanberry & Payne, 2023). One challenge is the fact that students returned to the classroom in person with less motivation as before, or not having the prerequisite required for this course because of learning interruption during online instruction. During the pandemic, instructors and students were forced to adapt into a remote learning model while combatting their fears and anxiety surrounding the COVID-19 disease (Ghazi-Saidi et. al, 2020). The academic and emotional toll this adjustment brought to instructors and students eradicated any form of motivation to adjusting back to traditional learning in the classroom if that ever happened at all.

The second challenge could be that most students are not familiar with the standards-based grading system. Students might feel compelled to only answer the questions on an assessment that will help them master a goal. Consequently, students might focus more on the number of problems they got wrong (or could not complete) instead of the number of goals they need to master on an assessment. More instructional time should be focused on breaking down the grading system to better prepare students for success in the redesigned Pre-Calculus course.

Overall, students were able to master goals related to linear equations and properties of functions (function class, composition of functions, etc.) by the end of the semester. These goals also required them the least amount of time to master. However, as in Nasir et al. (2017)'s findings, students in this study struggled mastering goals related to trigonometry, taking them the longest to master as well. Goals related to trigonometry should be introduced earlier in the course and include more opportunities for students to master it. Additionally, the course should allocate more time for new students to familiarize themselves with the standards-based grading system. These suggestions can further improve the quality of the redesigned Pre-Calculus course and, in the long run, increase the rate of student retention in STEM majors.

Limitations

A potential limitation of the study is not having insights from the students who took the redesigned Pre-Calculus in the Spring 2023 semester. Students overall struggled to master trigonometry concepts in the course, and Aminudin et al. (2019) aimed to provide reasons for why that was the case for their student population. A survey or interview would have affirmed the conclusions made by Aminudin et al. (2019), and perhaps share more insight that was not previously stated. There is an abundance of reasons as to why students struggled with this and other concepts in the course, ranging from academic to personal, and referring to temporary and permanent circumstances. Evidently, addressing every concern is not possible, but adjusting the course to meet the prevalent needs of students can enhance their experience and learning in the course. This paves the way for future research.

Future Research

Further research is needed for this study to improve student successes in a redesigned Pre-Calculus course. Future research includes surveys on students' insights on the concepts they found difficult and other challenges they experienced in the course. Students' feedback in the redesigned Pre-Calculus course would be detrimental to further address the challenges STEM students face daily.

Acknowledgements

The authors would like to acknowledge the financial support of the Office of Undergraduate Research 2023 Summer Research Award Program and the Mathematics Department of College of Natural and Behavioral Sciences at California State University-Dominguez Hills.

References

Aminudin, M., Nusantara T., Parta, I. N., Rahardjo S., As'ari, A. R., & Subanji, (2018). Engaging problems on trigonometry: why were students hard to think critically? *Journal of Physics: Conference Series.* 1-10.

Cho, P., Moore-Russo, D., & Stockton R. (2014). How Students Come to Understand the Domain and Range for the Graphs of Function. *Proceedings of the Joint Meeting 2 - 281 of PME 38 and PME-NA 36. 2,* 281-288. https://files.eric.ed.gov/fulltext/ED599786.pdf

Complex Instruction. Stanford University. (n.d.). https://complexinstruction.stanford.edu/

Friendly, Rayna. "Incorporating Reading Apprenticeship (RA) Into Your Courses – Nov. 2019 Teaching Community of Practice (TCoP) Recap!" *ECampus,* 3 Dec. 2020, https://blogs.sjsu.edu/ecampus/tag/readingapprenticeship/.

Frketic, A. M., Berk, D., & Rossi, T. (2019). An Investigation into College Students' Learning about Logarithmic Functions: A Thorny problem. *Psychology and Behavioral Science: International Journal, 10*(3), 1-4.

Ghazi-Saidi, L., Criffield, A., Kracl, C. L., McKelvey, M., Obasi, S. N., & Vu, P. (2020). Moving from face-to-face to remote instruction in a higher education institution during a pandemic: Multiple case studies. *International Journal of Technology in Education and Science* (IJTES), *4*(4), 370-383.

Goyer, A., Alison, A., & Wand, J. (2020). A Redesign of Precalculus at California State University, Monterey Bay. *Problems, Resources, and Issues in Mathematics Undergraduate Studies, 21*(3–5), 492–503.

George, W., & Gallagher, K. (2022). What in the World Just Happened? Students' Self-Regulation in Asynchronous PreCalculus During the COVID-19 Pandemic. *Journal of Humanistic Mathematics, 12*(2), 315–345. https://doi.org/10.5642/jhummath.KQUM2943

Jones, M. G., & Lanaghan, S. (2021). Building and Sustaining Success in Precalculus: A Multi-Pronged Approach. *Problems, Resources, and Issues in Mathematics Undergraduate Studies, 31*(9), 962-974.

Lampropoulos, G., & Admiraal, W. (2023). The Impact of COVID-19 Pandemic on Primary, Secondary, and K-12 Education: A Systematic Review. *International Journal on Studies in Education*, *5*(4), 348–440. https://doi.org/10.46328/ijonse.161

Mkhatshwa, T. (2021). An investigation of students' content understanding, perception changes, and experiences in a flipped precalculus course. *Association for University Regional Campuses of Ohio, 27*, 41-73.

Nasir, A. M., Omar, N. A. B., Sarudin, E. S. B., Masrom, S., & Ahmad N. H (2017). Teaching and Learning or Pre-Calculus: An Insights of Educators. *Journal of Fundamental and Applied Sciences, 9*(6S), 452-467.

Stanberry, M. L., & Payne, W. R. (2023). Teaching undergraduate calculus at an urban HBCU through a global pandemic. *International Journal of Education in Mathematics, Science, and Technology* (IJEMST), *11*(2), 340-357. https://doi.org/10.46328/ijemst.2557

Viera, J., Convertino, C., Mein, E., Villa, E. Q., & Hug, S. (2019). *Shifting Pre-Calculus from a Gatekeeper to a Gateway Course*. 2019 CoNECD - The Collaborative Network for Engineering and Computing Diversity. https://peer.asee.org/31790

Author Information

Angel Perez

https://orcid.org/0009-0009-6444-806X

California State University

Dominguez Hills

1000 E. Victoria Street. Carson, CA 90747

United States

Ha Nguyen

https://orcid.org/0000-0002-9940-7761

California State University

Dominguez Hills

1000 E. Victoria Street. Carson, CA 90747

United States

Contact e-mail: *hnnguyen@csudh.edu*

Appendix. Learning Goals in a Redesigned Pre-Calculus Course (Jones & Lanaghan, 2021)

Essential				Proficient			
Linear Functions							
1L			Be able to solve a linear equation or system of linear equations.	23L			Be able to model a situation involving linear motion with appropriate parametric equation(s) and interpret the solution.
2L			Be able to model a situation with appropriate linear equation(s) and interpret the solution.				
3L			Be able to determine the slope or equation of a linear function given its graph or a table of values.				
Quadratic Functions							
4Q			Be able to solve a quadratic equation.	24Q			Be able to determine the equation of a quadratic function given its graph.
5Q			Be able to model motion of objects falling with the force of gravity with appropriate quadratic equation(s) and interpret the solution.	25Q			Be able to model problem situations with an appropriate quadratic function equation(s) and interpret the solution(s).
6Q			Be able to determine and interpret the vertex of a quadratic function given an equation or context.				
Exponential Functions							
7E			Be able to solve an equation with an unknown exponent.	26E			Be able to use the definition and properties of logarithms to rewrite expressions involving logarithms in different forms.
8E			Be able to model a situation with appropriate exponential equation(s) and interpret the solution.				
9E			Be able to apply rules of exponents and radicals to simplify expressions.				
Function Concepts							
10F			Be able to determine inputs or outputs from a function table or graph	~~27F~~	~~-----------~~	~~----------~~	~~Be able to identify the intervals on which a given function is increasing or decreasing.~~
11F			Be able to determine a composition of functions given in any form (graph, table, equation).	~~28F~~	~~-----------~~	~~----------~~	~~Be able to compute the average rate of change of a given function on a given interval.~~
12F			Be able to perform arithmetic (sum, difference, product, quotient) on functions given in any form (graph, table, equation).	29F			Be able to determine an appropriate function class (linear, quadratic, exponential, trigonometric) to model a particular situation.
13F			Be able to determine the inverse of a function given in any form (graph, table, equation).	30F			Be able to solve equations involving rational functions.
14F			Be able to give the solution to an inequality or set of inequalities using proper mathematical notation.	31F			Be able to simplify and perform basic operations (+, -, *, /) on rational expressions.
15F			Be able to determine or describe a transformation (reflection, translation, dilation) of a function given in any form (graph, table, equation).				
16F			Be able to produce a graph of a given rational function, indicating the vertical asymptotes, and x- and y-intercepts, if any.				
17F			Be able to determine the domain and/or range of a function given as an equation or a graph.				
18F			Be able to identify functions and operations that could be combined to produce a given function.				

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	29	30	31	32	33	34	35	36
Q1	1	1	1							2													1											
Q2				2	1	1								1										1										
Q3						1		1		1				1											1		1							
Q4							1	1	1				1																					
Q5													1					1								1								
Q6															1	1	1												1					
Q7																					1	1										1		
Q8																					1	1								1				
Q9																			2	2														
Q10																			1			1											1	
E1	2	2	1	2	1	2		1	1		2	2		2			1	1					1	1	1		3							
E2	2	1	1	1	1	2			1	1	1	1	1	1	2	1	2	1			1		1	1	1	1	1	1	1			1		
E3	1		1	1	1	1	2	2	1	1	1	2	3	2	1	1	1	1	3	1	1	1		1	1	1	1	1	1	1	1	1	1	
E4							1	1	1	1	1	1	2	2	1	1	2	1	4	2	1	1		1		1	1	1	1	1	2		2	2
FE	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1					1		1	1	1	1	2
	7	**5**	**5**	**7**	**5**	**8**	**5**	**7**	**6**	**7**	**6**	**7**	**9**	**10**	**6**	**5**	**8**	**6**	**11**	**6**	**6**	**6**	**4**	**5**	**4**	**4**	**7**	**4**	**4**	**4**	**4**	**4**	**5**	**4**

Trigonometric Functions							
19T			Be able to solve for an unknown angle and interpret the result in the appropriate quadrant.	32T			Be able to determine the equation of a trigonometric function given its graph.
20T			Be able to model a situation involving motion on a circle with appropriate trigonometric parametric equation(s) and interpret the solution.	33T			Be able to solve problems involving the law of sines or law of cosines.
21T			Be able to use the distance formula or the equation of a circle in context.	34T			Be able to determine the length of an arc of a circle or the area of a sector of a circle.
22T			Be able to determine a missing angle or side in a right triangle.	35T			Be able to prove trigonometric identities.
				36T			Be able to solve trigonometric equations by first transforming the equation using trigonometric identities.