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Chrysanthi Skoumpourdi
Professor, University of the Aegean, Rhodes, Greece

Antonia Matha
M.Sc., University of the Aegean, Rhodes, Greece

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Abstract

Teaching and learning of mathematics, due to their abstract nature, are enhanced, especially at an early age, using educational materials. The wide variety of the available math's educational materials requires teachers to evaluate them in order to incorporate them to their teaching practice. Contributing to this field, the purpose of this paper is dual. Firstly, it intends on defining the factors that could be included in a framework for evaluation of math educational material. Secondly, it aims on using this framework to evaluate specific educational materials that are used for the construction of early number concept. The results showed that the factors that could compose a framework of evaluating math educational materials could be related with 1. Evaluation of the material itself, independently of its context of use, 2. Evaluation of the material in the social context of its use, as well as 3. Evaluation of materials' acceptability to the general educational community. From the evaluation of specific educational materials that are used for the construction of early number concept it seemed that no material itself could be considered, as suitable for teaching all the individual constructs of the number concept in early years' mathematics, according to the developed framework.

Introduction

Teaching and learning of mathematics, due to their abstract nature, are enhanced, especially at an early age, through the use of educational materials. Educational materials can be curriculum recourses, as well as outside curriculum materials. The former are conventional educational materials, such as school textbooks, that in many countries are provided to teachers and are often the sole means for mathematics teaching at school (Skoumios & Skoumpourdi, 2018; Weiss, 1997).

The latter are no conventional educational materials, but artefacts that are designed for specific mathematical purposes, serving as external representations of mathematical concepts and are used as auxiliary means for teaching and learning mathematics (Golafshani, 2013; Meira, 1998). The wide variety of these outside curriculum materials requires teachers to devote time in researching, selecting and evaluating them in order to incorporate them to their teaching practice. This is a demanding process and for this reason teachers express the need for a framework that could help them evaluate these materials (Skoumpourdi, 2012). However, although research highlight the positive contribution of educational materials in effective teaching and successful learning of mathematics (Marshall & Swan, 2008), there is insufficient research on the evaluation/selection of these materials.

Contributing to this field, the purpose of this paper is dual. Firstly, it intends on defining the factors that could be included in a framework for evaluation of math educational material in order to be used from educators in their practice, as well as from researchers as a methodological tool. Secondly, it aims on using this framework to evaluate specific educational materials that are used for the construction of early number concept in order to describe the framework's way of use. The research questions posed were the following:

1. Which factors could compose a framework of evaluating math educational materials?
2. Could spike abacus, Pascaline, base-ten Dienes blocks, numicon and sumblox be considered, as suitable educational materials for representing and teaching number concept in early years' mathematics, according to the developed framework?

Theoretical Framework

Early Number Concept

Perception of the number concept is a key objective for the early years' mathematics education. The number-building model is based on counting, because children are familiar with numbers and operations from a very young age (Steffe & Cobb, 1988). Counting is defined as an action that involves sequencing the numbers in the correct order and connecting a number to a single element of a collection (Fuson & Hall, 1983). Subitizing, that is the direct estimation of a quantity, is considered to be an important complementary ability that promotes knowledge on the number concept (Clements, Sarama, & Mac Donald, 2018). Number symbols recognition and writing, ordering numbers in the conventional sequence, creation and comparison sets of objects collections, counting and ordering objects, addition and subtraction, as well as place value in decimal system, are also integral parts of the number concept in early years' mathematics (NGACBP & CCSSO, 2010).

Educational Materials in Teaching and Learning Mathematics

According to an increasing number of researchers internationally, outside educational materials, as external math concept representations, are essential elements in effective teaching and successful learning of mathematics (Gueudet, Pepin, & Trouche, 2013; Meira, 1998; Remillard, 2013). When children are encouraged to use educational materials in a way that makes sense to them, they actively engage in the teaching process (Tran, 2015), they achieve deeper conceptual understanding of mathematics concepts and improve their mathematical performance (Liggett, 2017; Swan & Marsall, 2010), they develop confidence and flexibility in their problem solving ability (Jacobs & Kusiak, 2006), they cultivate new learning strategies, mathematical way of thinking, computational skills, critical thinking and creativity (Golafshani, 2013). There are many advantages of using outside educational materials for constructing early number concept, such as reducing the cognitive effort demanded, helping children develop advanced counting strategies, create a mental representation of number symbols, and operate with numbers in a flexible way (Manches & O'Malley, 2016; Obersteiner, Reiss, Ufer, Luwel, & Verschaffel, 2014). Number concepts' content, structure, and organization, as well as numerical cognition are influenced by the materials used to represent and manipulate them (Overmann, 2018).

Evaluation of Educational Materials

Early childhood educators search for outside educational materials on various sources such as the internet, their own libraries, newsletters, etc., and they adapt them to their teaching practice if they evaluate them positively (Davis, Janssen, & Van Driel, 2016). The criteria they use to evaluate them are in accordance with their students' needs (Son & Kim, 2015), their own instructional goals (Brown, 2009; Remillard, 2013), their teaching practice (Janssen, Westbroek, & Doyle, 2015), as well as their experience with them (Sherin & Drake, 2009). From the materials they evaluate positively, they usually select those educational materials that require minimum adaptation (Brown, 2009; Remillard, 2013; Roehrig, Kruse, & Kern, 2007). In addition to the above subjective criteria, they also take into consideration the alignment of outside educational materials to the mathematical concept (Skoumios & Skoumpourdi, 2018), to the curriculum recourses (Davis et al., 2016), as well as to the general educational context (Roehrig et al., 2007). Studies also confirm that other criteria, such as material's availability, accessibility and affordability are crucial in teachers' final decisions (Skoumios & Skoumpourdi, 2018). Transparency, as the main factor of outside educational materials evaluation, which is highlighted in theoretical considerations, is not mentioned by empirical research as a teachers' criterion of evaluation.

Transparency is a multidimensional concept defined through the description of its characteristics (Chase & Abrahamson, 2013) and can be achieved/discovered gradually (McNeil & Jarvin, 2007; Meira, 1998). Transparency is affected by material's dual representation; the simultaneous existence of the artifact and the mathematical symbolism within it (Uttal, Amaya, del Rosario Mait, Cohen, O 'Doherty, & DeLoache, 2013). Each mathematical educational material consists of specific characteristics and aims at teaching a specific mathematical concept. If the quality of the correspondence between the material and the mathematical concept it represents is positive (Meira, 1998) and the bond of its characteristics with the mathematical concept is strong and wide, or else if a material is directly linked to the concept and is able to cover all its aspects without leading to misunderstandings, then it is considered scientifically valid (Stacey, Helme, Archer, & Condon, 2001). Thus, one of the main transparency's characteristics concerns the 'epistemic fidelity' of the material. The epistemic fidelity takes place regardless the social context of its use.

A second characteristic of transparency concerns ‘form transparency’ meaning transparency of material’s parts (Chase & Abrahamson, 2013). In other word, whether the parts of the material are visible to users and can be used. ‘Form transparency’ does not necessarily mean understanding of the logic behind material’s use (McNeil & Jarvin, 2007). A third characteristic concerns ‘operation transparency’ that means understanding the logic behind the use of the educational material (Chase & Abrahamson, 2013; McNeil & Jarvin, 2007) and is related to the social and cultural context in which the material is incorporated and used (Meira, 1998). Transparency, in this sense, is defined as a process, as a way of engaging that allows users to discover the internal mechanism of a material. More specifically, if a material allows users to discover its internal operating mechanism, then its operation is considered transparent (Stacey et al., 2001).

A fourth characteristic of transparency concerns ‘cognitive validity’. Cognitive validity depends on the power of the connection between material’s use and users’ cognitive processes, which means whether mental processes emerge from the use of the material (Zbiek, Heid, Blume, & Dick, 2007). Cognitive validity can only be evaluated through the investigation of the users’ representations. A fifth characteristic concerns accessibility. According to Stacey et al. (2001), a material should be evaluated when used by large number of users in order to examine whether it is accessible and likeable, able to cause discussion and engage users. Accessibility refers, not to specific users in a particular social context but, to general population. In other words, it refers to a collection of social and psychological factors that occur during its educational use and affect the totality of users (McNeil & Jarvin, 2007).

Methodology

Developing a Framework for the Evaluation of Math’s Educational Materials

Taking into consideration the above research data regarding the criteria that teachers use to evaluate educational materials, as well as, the theoretical positions related to the transparency as the main factor for educational materials’ evaluation we developed the Framework for Evaluating Math’s Educational Materials (FEMEM). This initial form of the FEMEM is developed on six axes, with the three of them related to objective evaluation and the other three to subjective evaluation (Figure 1). In this paper only the three objective evaluation axes will be discussed.

The FEMEM’s three axes of objective evaluation are focused on (see Figure 1): 1. the evaluation of the material itself regardless of its context of use, 2. the evaluation of the material in the social context of its use, as well as 3. the evaluation of the material’s acceptability to the general educational community. These three axes could be the main factors that teachers and researchers should take into consideration before selecting/proposing and use/analyse educational materials. Each of these three axes is analyzed into sub-axes.

More specifically, the first axis of the FEMEM is focused on the evaluation of the educational material itself in order to investigate the correspondence between the material and the mathematical concept it represents. In this axis, the material is evaluated, independently from its context of use. The evaluation takes place through the investigation of the degree of: 1a. The coverage of the mathematical concept, that means, whether the material covers the mathematical concept’s constructs. 1b. The material’s epistemic/mathematical fidelity, that means, whether the material represents ‘correctly’ the mathematical concept’s constructs, and 1c. The degree of visibility, that means, whether the parts of the material are visible and can be used. Thus, for evaluating outside educational material itself, the following questions must be answered: i) Does the material cover the mathematical concept’s constructs? ii) Does the material have mathematical fidelity for this concept? iii) Are the material’s parts visible and can be used (see Figure 1)? This first axis is based on the first dimension of the double frame of reference of transparency (Chase & Abrahamson, 2013; McNeil & Jarvin, 2007; Meira, 1998; Stacey et al., 2001), that is the quality of the correspondence between material and mathematical concept it represents, and it is of the utmost importance if we assume that the aim of an educational material is to represent a specific mathematical concept to be taught.

The second axis of the FEMEM is focused on the evaluation of the material in the social context of its use. In this axis is evaluated the material’s usability in practice. The evaluation takes place through the investigation of the degree of: 2a. users’ understanding of material’s operation, that means, whether users understand the material’s mechanism and can manipulate it accordingly, and 2b. users’ cognitive process activation, that means, whether there is a relationship between material’s aims and mental processes elicits from users. User’s mental processes can be basic (as sensation, attention, and perception) and complex (memory, learning, language use, problem solving, decision making, reasoning, and intelligence). For evaluating the educational material, in the context of its use, the following questions must be answered: i) Is material's operations understandable by users? ii) Does the material activate user's cognitive process (see Figure 1)? This second axis is based on the second dimension of the double frame, that of the evaluation of the material in the social context of its use, including Zbiek et al' (2007) parameter of cognitive validity, and is a very important axis since teachers' goal is to activate students' cognitive process through the use of a material with understandable operation.

The third axis of the FEMEM is focused on the evaluation of the material's acceptability to the general educational community and evaluates the degree of: 3a. the material's availability to potential users, that means whether the material can be found relatively easily (e.g. in the market). 3b. the material's accessibility to potential users, that means whether the material is used by many users, and is familiar and likeable to them, and 3c. the material's affordability, that means whether a quantity of the material could be bought for use in the classroom. Thus, for evaluating the material's acceptability to the general educational community, the following questions must be answered: i) Is the material available? ii) Is the material accessible? iii) Is the material affordable (see Figure 1)? This third FEMEM's axis, is grounded on Stacey' material's accessibility (2001), as well as on research results that confirm that factors like material's availability, familiarity and cost are crucial in teachers' evaluations (Skoumios & Skoumpourdi, 2018). A material that is not available, accessible and affordable, although it may have been evaluated positively in the first two axes, it cannot be used in the classroom.

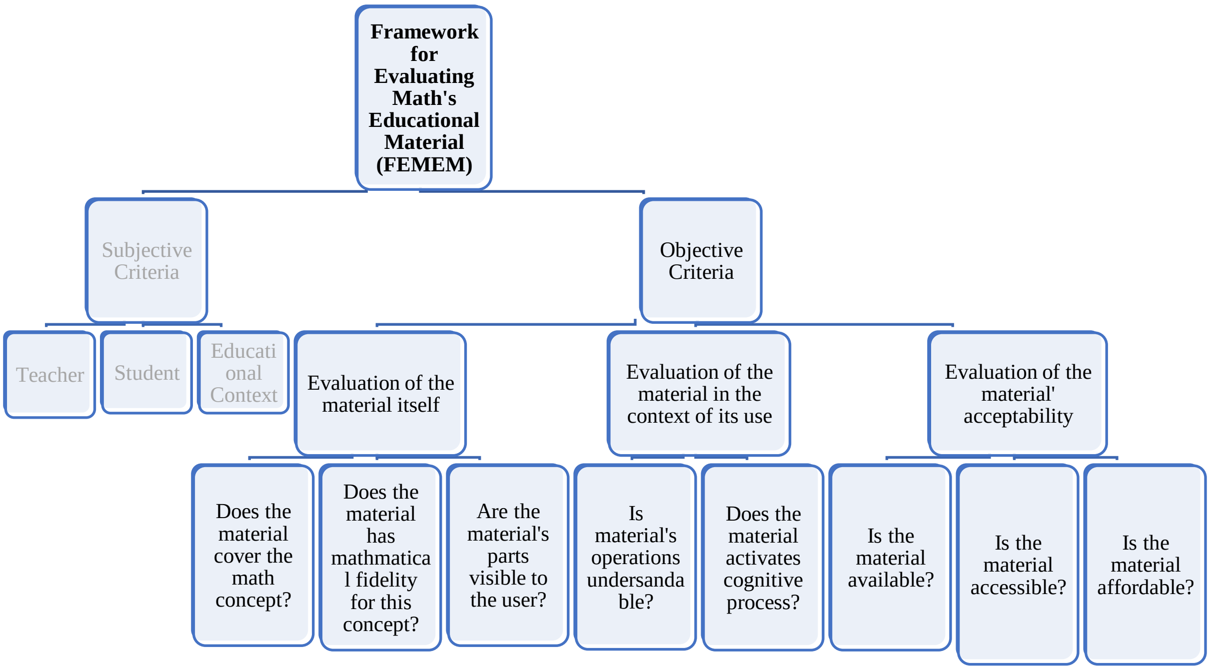

Figure 1. Framework for Evaluating Math's Educational Material (FEMEM)

Procedure

The developed FEMEM was used in order to evaluate five educational materials for their suitability of potential use in the teaching practice and their role in activating mathematical thinking and constructing early number concept. Each material was evaluated using the three FEMEM's objective evaluating axes and sub-axes and was classified as: a. Unsuitable: in this category we classified the materials that took negative evaluation at least in 1b, or/and 3a, or/and 3b, or/and 3c. Negative evaluation in 1b means that the material does not represent 'correctly' the mathematical concept's constructs, and for this is not suitable for educational use.

Additionally, negative evaluation in 3a, 3b or 3c means that even the material is suitable for educational use, because of its unavailability, not accessibility or not affordability it becomes unsuitable in practice. b. Partly suitable: in this category we classified the materials for which the evaluation was positive for at least 1b, 3a, 3b, and 3c. Both in this category (partly suitable) and in the previous one (unsuitable) the negative evaluation of the rest sub axes (1a, 1c, 2a and 2b) does not affect the material's classification. Negative evaluation of 1a which means that the material does not cover the entire mathematical concept's constructs, does not show inefficiency of the material. It may be a specialized material that covers a specific construct of the concept. Negative evaluation of 1c, 2a and 2b show a material that its parts are not (easily) visible, its operations are not understandable and does not activates users' cognitive process. But these sub-axes (1c, 2a and 2c) can be achieved through the teaching practice. c. Suitable: in this category we classified the materials that were evaluated positive for all (eight) sub axes.

The Selected Educational Materials

The five educational materials selected for evaluation were, spike abacus, Pascaline, base-ten Dienes blocks, numicon and sumblox. These materials are physical manipulates that are proposed by researchers to be used for teaching early number concept constructs such as counting, creating and comparing collections of objects, subitizing, recognition and writing of number symbols, counting and ordering objects, operations of addition and subtraction, as well as place value in decimal system. Additionally, there is research data for their educational use.

Specifically, spike abacus has a long history in human civilization, and this is why it appears in the market in a variety of forms, depending on the era and the locus. Spike abacus is the most known form in Europe. It consists of two to ten vertical equal-sized rods, fastened on a base, with the upper end free and with beads in two to ten different colors respectively. The rods, from right to left, represent ones, tens, hundreds etc. The rods' length is equal the length of 9 beads (see Figure 2). Spike abacus also occurs in other forms, in which either the length of the rods differs, or the upper end is not free (Skoumpourdi, 2009) (see Figure 3 and 4).

Figure 2-4. Different Forms of Spike Abacus

The Pascaline is a later form of Blaise Pascal's original engine, which was created around 1642-65. Its current and simplified form consists of 10-unit disks, according to decimal system. For early years' mathematics, Pascaline is preferred with three main discs in which numbers up to 999 can be imprinted. These three discs, A, B and C, are next to each other and they communicate through discs D and E. All disks move clockwise. When disc C completes a full rotation (10 clicks), the purple pin pushes disc B by one position (1 click equals 1 ten). Similarly, Disc A moves by one position (1 click equals 1 hundred) when there is a full rotation by disk B. Before any calculation, all three disks must be positioned in such a way so that the red triangles point to zero. Then, after the necessary moves the three red triangles will show the result of the calculation (see Figure 5).

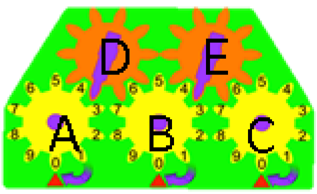

Figure 5. Pascaline

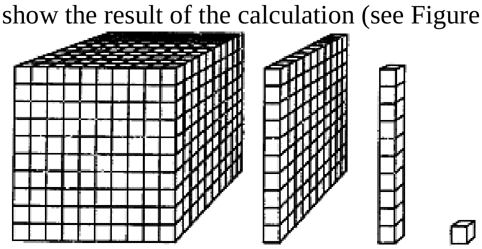

Figure 6. Base-Ten Dienes Blocks

The base-ten Dienes blocks (from now on referred as Dienes blocks) are linked to the theoretical foundation of materials' use in mathematical education. Dienes blocks are cubes compositions and contain cubes of one centimeter as ones, rods of ten cubes as tens (1X10), plates of hundred cubes (10X10) (assuming a thickness of zero) as hundreds, and big cubes of smaller cubes (10X10X10) as thousands (see Figure 6). Their purpose is to represent the decimal numbering system and the concept of base, but they are also used for addition and subtraction. The numicon is derived from the development of mathematical education, as a designed educational material for special educational purposes. It consists of ten plastic multicolored plates with as many holes as the number they represent (see Figure 7), based on the pattern of numbers. Their special feature is that they can represent two-digit numbers as a sum of ten(s) and ones.

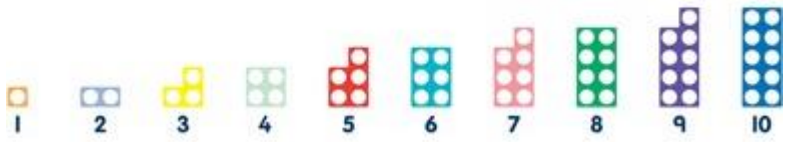

Figure 7. Numicon

The sumblox is a modern, promising, math educational material advertised online. It consists of wooden bricks in the form of number symbols. The bricks start from number 1 to number 10. All other numbers can be represented as combinations of the above. The height of each brick is 1.5 cm higher than the brick of the immediately smaller number. As a result, when a brick is placed on top of another, their height is equal the height of the brick that shows their sum (see Figure 8). The presentation of the numbers in their symbolic form, as well as the fact that the heights of the bricks augment gradually, distinguishes sumblox from other educational materials used for addition and subtraction constituting a "bridge" from the written form of an operation to its understanding. The complete sumblox package (see Figure 9) –as it is available also in smaller edition- consists of thirty bricks of number one, twelve bricks of number two, eight bricks of each of the numbers three, four, five, six, seven, eight and nine and two bricks of number ten. The material is accompanied by manuals with explanation of its use and suggested activities.

Figure 8. Addition With Sumblox

Figure 9. Sumblox Package

Other materials, such as arithmetic rack, number line, Montessori material, Cuisenaire rods, fraction tiles, bug counters, decadots, twenty-frame, hundred-square etc., that are also related to number concept constructs, were not selected to be evaluated because of their similarity with the selected materials, because of their unsuitability for early years' mathematics or because of the absent of research data for their educational use.

Results and Discussion

The presentation of the results takes place with the analysis of each of the educational material based on the three framework's objective axes and sub axes. At the end of the analysis of each material the level of its appropriateness is deduced.

Spike Abacus

Regarding the evaluation of spike abacus itself, it seems that its parts are visible and can be used by the potential users. Spike abacus is proposed for creating, comparing, counting, estimating and ordering objects and collections of objects, as well as for addition and subtraction. Its main characteristic is the representation of place value. It represents the realistic form of physical numbers, imprinting through the rods the positions of the digits giving them the corresponding value. Spike abacus covers these mathematical concepts' constructs with mathematical fidelity (Skoumpourdi, 2009). On the contrary, it is not suitable for recognizing and writing the number symbols, because number symbols are not presented on the artifact.

Regarding the evaluation of spike abacus in the social context it seems that the users without knowledge of its use, make collections of beads on different rods and calculate them either through counting or through direct estimation, compare collections of objects depending on the arrangement of beads or rods, but also calculate the total beads by adding or subtracting them (Skoumpourdi, 2012). Through the movement of the beads one could observe users' actions in the artifact and thus (partly) understand their process of thinking. However, the basic function of spike abacus, that is, the place value, cannot be understood by a novice user, as it presupposes both the knowledge and the acceptance of the convention that the right-to-left rods represent the position of ones, tens, hundreds, etc (Gravemeijer, Cobb, Bowers, & Whitenack, 2000). Spike abacus does not facilitate the link between their actions and the construction of the intended intellectual representation of the concept (Miller & Stigler, 1991). The process of replacing ten beads of a rod with one bead in the left-hand rod is not immediately understood by novice users. This process is a social construction based on the written and in many cases oral form of numbers. This difficulty may be avoided through appropriate activities, which can, in the long run, convert spike abacus into a mental tool that can be used even for difficult calculations, but also into a "semantic" material that links the realistic to the semantic form of numbers (Bartolini, Corni, Mariani, & Falcade, 2012). However, the actions of experienced users with the artifact can be linked with the function of the artifact and the learning objective to construct the number concept. The historical and cultural imprint of spike abacus at different eras and cultures around the world, as well as its presence in the Greek kindergartens and the Greek primary school math's textbooks, prove its wide availability and accessibility (Skoumpourdi, 2009). The above mentioned with the issue that spike abacus is an affordable material ensures its positive evaluation regarding its acceptability to the general educational community.

In conclusion, it becomes apparent that spike abacus, in its classical form, is a math educational material which may be considered partly suitable for teaching the number concept in early years' mathematics. It covers most of the number concept constructs and represents them strongly, that means number concept constructs are represented with mathematical fidelity. But number concept constructs, such as place value are not always represented directly, in a way that is obvious to novice users. Its parts are visible to users, but its operation is not always evident to novice users, as they cannot discover it on their own. However, its effect in practice is positive for experienced users, through specially designed teaching interventions, activating their cognitive process. Furthermore, spike abacus is an acceptable math educational material to the general educational community because it is an available, accessible and affordable material (see Table 1).

Pascaline

Regarding the evaluation of Pascaline itself, it seems that is a mathematical educational material that represents the number symbols in their order, the place value of digits, as well as addition and subtraction as concepts, rather than as calculations (Bartolini & Boni, 2009). Pascaline is directly and strongly linked to the recognition of the sequencing of the numbers in the correct order, as it is obvious which number follows or proceeds in clock- or anti-clockwise direction. Pascaline can be used to calculate sums and differences, without requiring any mental process and through these actions the concept of place value could also emerge. On the contrary, due to the fact that it represents numbers as arithmetic symbols on disks and not as "entities", it does not serve as a material for creating, counting, subsiding, comparing and ordering objects and collections. Ordering objects can only be seen, through the position of the three disks (1st, 2nd, and 3rd). From the above mentioned it seems that Pascaline represents only some of the constructs of the early number concept, but it covers them with mathematical fidelity. Its parts are visible and can be used by the potential users.

A research about Pascaline's influence, both as physical and digital math educational material showed that although Pascaline allows users to get involved, it does not make it easy for them to understand its function. That conclusion derived from the calculation of mathematical operations by 6 years old primary school students through a 12-week observation of teaching practices (Soury-Lavergne & Maschietto, 2015). The main findings of the research were that Pascaline is not easily understood for its use without prior instruction and that students find difficult splitting numbers into ones and tens in order to use Pascaline more efficiently. Another study (Maschietto & Ferri, 2007; Maschietto & Soury-Lavergne, 2013) that investigated whether Pascaline can act as a mediator of understanding addition and subtraction, showed that Pascaline's operating mechanism was understood by all 9 years old students (in different timings depending on the abilities). Students also conceived Pascaline as a computational tool. That became apparent through their success both in composing Pascaline's manual and in expressing two different ways of addition using mathematical terminology. In other words, users faced Pascaline more as a computational tool, such as a calculator, rather than as an instrument that may be useful for discovering new knowledge. In conclusion, regarding the evaluation of Pascaline in the social context it seems that Pascaline's operations are understandable only after teaching to older students and that the structure of this artifact shows users' actions through which their thinking process could be hypothesized as long as their actions reflect their cognitive processes.

Regarding its acceptability to the general educational community, it seems that Pascaline is a math educational material that is not widely known nor used. However, is available on the online market and its cost is affordable. To sum up, from all the above, it seems that Pascaline is partly suitable material for the perception of number concept constructs in early years' mathematics, as it only represents specific number concept constructs. It also appeared to be elusory to novice students or used as calculation tool. Furthermore, it is not accessible to the general population, but it is available and affordable (see Table 1).

Base-ten Dienes Blocks

Regarding the evaluation of Dienes blocks itself; it seems that is a mathematical educational material that represents decimal numbering system and the concept of base. If the cubes can be connected and disconnected, Dienes blocks can be used for addition and subtraction, for creating objects' collections, ordering them and calculating the number of cubes, that constitute them, through counting, comparing, and subsiding. But Dienes blocks are not suitable for the identification and writing of number symbols or the sequencing of numbers in the correct order. Although the structure of Dienes blocks seems to be visible, as its parts are broken into units, many students find it difficult to identify the "inner cubes" that are not visible from the outside. Dienes blocks cover the majority of the constructs of the early number concept, with mathematical fidelity but its parts are not always visible to the potential users.

Regarding the evaluation of Dienes blocks in the social context it seems that for young users has either limited positive effects or is unable to improve the mathematical perception of adding and subtracting numbers. Research has shown that the use of Dienes blocks seemed to have little positive influence on improving two-digit number subtraction calculations (Resnick & Omanson, 1987), whereas the use of Dienes blocks for calculating two-digit numbers helped students gain computational skills (Labinowicz, 1985). Dienes blocks significantly limit the conversion of calculations to intelligible computations due to the insufficient mental representations created by the use of the material from grade 2 students (Beishuizen, 1993). In an earlier research Beishuizen also observed that blocks due to their form reduce the understanding of the (formal) N10 strategy (splitting the second addend into tens and ones), while enhancing the (informal) 1010 strategy (splitting both addends into tens and ones and adding them respectively). Nevertheless, their use for numbers over a thousand gets difficult because the volume of the material increases abruptly. The structure of the artifact partly permits the emergence of users' thinking process, since their actions cannot fully embody their cognitive processes (Kurumeh, Chiawa, & Ibrahim, 2010). On the other hand, research showed that for addition and subtraction operations, their effectiveness is evaluated through students' performance in finding new algorithms for operations - especially in multidigit numbers (Olive, 2008). Dienes blocks promote understanding of place value due to the need of regrouping while adding and subtracting. This ability seems to be well established even after the use of the material, through students' verbal or written expressions of numbers (Fuson & Briars, 1990).

Regarding Dienes blocks' acceptability to the general educational community, it seems that it is an available educational material because it can be found easily in the market. It is also accessible to potential users because it appears as representation in many school textbooks of almost all primary school levels, as well as due to its use internationally (Skoumpourdi, 2012). Taking into consideration the above mentioned with the fact that Dienes blocks is an affordable material ensures its positive evaluation in this axis. Based on all the above, Dienes blocks is an educational material that is characterized as partly suitable for teaching the number concept, as, although it is linked strongly and directly with the most number concept constructs, its social context seems to be more compatible with older students. Finally, the ease of supply of the material and the fact that it is available to users and affordable, are supplementary advantages of the material and make it acceptable math educational material to the general educational community (see Table 1).

Numicon

Regarding the evaluation of numicon itself, it seems that the structure of this educational material is suitable for counting, comparing, ordering and estimating quantities. It is suitable for understanding the digits place value through the gradual completion of each row by two "holes" and then the addition of one more "hole" of above which reveals the order of numbers and represents the digits place value and the creation of numbers (Dowker, 2008). Numicon's structure also emphasizes the importance of number patterns and promotes extended knowledge, such as fact families of numbers over 10, representing two-digit numbers as a sum of ten(s) and ones, as well as its ability to make odd numbers, even numbers, and doubles apparent (Gifford, Griffiths & Back, 2017). But numicon is not suitable for recognizing and writing the symbols of numbers and for understanding the order of numbers. It is also not suitable for the creation of object collections, as it consists of compositions of predefined collections and not of sum of ones.

Regarding the evaluation of numicon in the social context it seems that research results are limited and differ since numicon has recently been introduced, mainly in Europe, into the teaching practice for both general and special education children. There is research (Hughes, 2006) with 5-6-year-old students concluded that due to numicon's structure, which presents numbers visually, relationships between numbers are apparent and number pattern recognition is facilitated. Another study found that students who had difficulty in learning found the material useful, while many of the capable students found it irrelevant and did not want to use it (Skevington, 2016). Other study concluded to three key conclusions (Jenkins, 2013): 1. The relationship between the use of numicon and students' ability to quantify numbers is weak, and this relationship is independent of their initial performance 2. There is no evidence linking the use of the material with the ability to correspond quantitatively and numerically to numbers. 3. The use of the material - especially by lower-performing students- led to a significant improvement in the understanding of the number concept. From all the above, it seems that the way numicon is handled varies, with correspondingly varying results. However, it seems that in order to be used for mathematical purposes it is necessary that teachers explain its way of use.

Regarding numicon's acceptability to the general educational community, it seems that it is an available educational material, because it can be found easily in the online market and its cost is low. However, it is rarely used in general education. It is mainly used in the field of special education. Therefore, it is familiar to a smaller proportion of educational population.

From the above, it seems that numicon is an educational material that is characterized as partly suitable for teaching early number concept, because is linked directly—in a way obvious to users—and strongly—with mathematical fidelity—with just few of the number concept constructs. Material's operations could be understandable by potential users and cognitive process could be activated but not always successfully without teaching. Finally, it is partly acceptable math educational material to the general educational community because although it is available, its cost is low and its supply is easy especially through on-line market, it is neither known nor familiar to the general population limited to special education (see Table 1).

Sumblox

Regarding the evaluation of sumblox itself, it seems that its structure is visible to the users and is suitable for recognizing number symbols, as well as the order of numbers since bricks are arranged by length—there is a difference in the height of the bricks depending on the digit they represent. It is also suitable for addition and subtraction, on condition that unlimited bricks are available. But addition and subtraction are treated solely as sum and difference respectively and not as removal of units. Numbers are only represented as sums of ones and tens and not as two digitals. Sumblox does not facilitate counting, creation of object collections, direct estimation of quantities, comparison of quantities, and perception of digits place value. It represents only few of the number concept constructs, such as number symbols and their order, with mathematical fidelity.

Regarding the evaluation of sumblox in the social context it seems from a unique research (Matha & Skoumpourdi, 2018) that it allows users' actions to be linked to the written and then mental processes, to the perception of numbers and their relationships, as well as to addition and subtraction. It seemed that although sumblox operations were understandable, it failed to highlight users' cognitive processes as they used it mechanically without critical prospective and reflection (Matha & Skoumpourdi, 2018). This reduces the cognitive validity of the material as it does not enable students develop their thinking. Sumblox seemed to be interesting only for low-performing students whereas for others it was considered difficult and time-consuming and they preferred to perform additions and subtractions with mental processes (Matha & Skoumpourdi, 2018).

Regarding sumblox's acceptability to the general educational community, it seems that it is twofold, with its advantages focusing on the worldwide availability of the material through its online purchase, but its disadvantages focusing on the high cost of purchasing and transporting it, and on that it is not known (Matha & Skoumpourdi, 2018). According to the above, sumblox is classified as an unsuitable math educational material because although it represents some constructs of number concept, such as number symbols, addition and subtraction, with which is directly related with, it offers limited possibilities to users to activate their cognitive process. The material becomes obsolete for calculations with numbers greater than twenty. Its usability in practice has clearly not yielded positive results, either in terms of learning outcomes or users' preference, but additional research is needed to confirm the above results. Sumblox's acceptability to the general educational community is not positive as it is a material still unknown to the general population and can only be purchased online at a very high cost (see Table 1).

Comparative presentation of the degree of suitability of the above materials in accordance to the axes defined is presented in Table 1.

Table 1. Suitability of Educational Materials for Number Concept Constructs in Early Mathematics (**** direct—in a way obvious to users—and strong—with mathematical fidelity—connection, *** strong but not direct connection, ** direct but not strong connection, * no connection, ✓positive, X negative, - no data)

			Spike abacus	Pascaline	Dienes blocks	Numicon	Sumblox
Evaluation of the material itself	Coverage of constructs of the number concept and mathematical fidelity	Counting	****	****	****	****	*
		Object Collections (create and compare)	****	*	****	***	*
		Subsiding	****	*	****	****	*
		Numbers (symbols, sequence)	*	****	*	*	****
		Ordering objects	****	****	****	****	*
		Addition	****	***	***	***	***
		Subtraction	****	***	***	***	***
		Place value	****	****	****	***	***
	Material parts visibility		****	****	**	****	****
Evaluation of the use of the material	Understanding material's operation (Novice users)		X	X	X	✓	✓
	Understanding material's operation (Experienced users)		✓	✓	✓	✓	✓
	Cognitive activation (Novice users)		X	X	X	✓	✓
	Cognitive activation (Experienced users)		✓	✓	✓	X	X
Evaluation of material acceptability	Available		✓	✓	✓	✓	✓
	Accessible		✓	X	✓	✓	X
	Affordable		✓	✓	✓	✓	X
Objective Evaluation	Suitability		Partly suitable	Partly suitable	Partly suitable	Partly suitable	Un suitable

Conclusion

The need of evaluating the outside curriculum educational materials that are used for teaching and learning mathematics led to the development of a framework for their evaluation. The developed FEMEM is a framework that could be used for evaluating math educational materials according to their suitability for potential use in the teaching practice and their role in activating mathematical thinking and constructing mathematical concepts. FEMEM incorporates subjective and objective criteria. The objective criteria that the current paper was focused are developed in three main evaluation axes: 1. evaluation of the material itself, independently of its context of use, 2. evaluation of the material in the social context of its use, as well as 3. evaluation of materials' acceptability to the general educational community.

The evaluation of the five educational materials—spike abacus, Pascaline, base-ten Dienes blocks, numicon and sumblox—based on the FEMEM, showed that no material itself could be considered, as suitable for teaching all the individual constructs of the number concept (counting, creation and comparison of object collections, subitizing, identification and writing of number symbols, ordering, addition, subtraction and place value), but some parts of it. These materials, although seemed appropriate and impressive for developing the number concept with mathematical fidelity, when evaluated lacked in cognitive validity or in factors of acceptability. In general, from the FEMEM it seems that the materials that represent with mathematical fidelity most of the number concept constructs, that their operations are understandable and activate user's cognitive process are suitable for use in educational practice in order to construct number concept. Acceptability, as an indicator of availability, accessibility and cost, ensures that these materials can be found and used in the classroom.

Evaluation results could be considered in order to select those educational materials the combination of which covers all the constructs of the number concept. It also provides elements that could be taken into consideration in order to design and develop more sophisticated and synthetic forms of educational materials, as multi-materials, that could be used to construct number concept. But more trials of FEMEM are needed in order to be refined and to be more effective in evaluating math educational materials. Furthermore, analysis is required to the other three FEMEM' axes, that of subjective evaluation, which are related with the broader educational context, as well as with teacher's and student's individual factors.

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Author Information

Chrysanthi Skoumpourdi
https://orcid.org/0000-0002-0492-2390
Professor
Department of Pre-school Education and Educational Design
School of Humanities
University of the Aegean
1 Dimokratias Ave., 85132 Rhodes
Greece
Contact e-mail: *kara@aegean.gr*

Antonia Matha
https://orcid.org/0000-0001-5878-9735
M.Sc.
Department of Pre-school Education and Educational Design
School of Humanities
University of the Aegean
1 Dimokratias Ave., 85132 Rhodes
Greece