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Fostering Creative Problem-Posing Skills of Pre-Service Mathematics Teachers within the Framework of Realistic Mathematics Education

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Abstract

In this study, we propose integrating creativity into problem-posing studies to ensure both measurability and conceptual integration. We highlight that creative activities, particularly those rooted in Realistic Mathematics Education (RME) and which promote real-world engagement, contribute to the development of pre-service mathematics teachers' (PMTs) problem-posing skills. We systematically examined the problem-posing competencies of students in relation to their creative processes, focusing on fluency, flexibility, and originality, which are considered core components of creativity. The study adopted both qualitative and quantitative methods; while the problem-posing processes of participants were examined qualitatively, the fluency, flexibility, and originality scores of the posed problems were evaluated quantitatively. The study involved 40 undergraduate students enrolled in the Problem-Solving in Mathematics Teaching course. Participants who were selected using the criterion sampling method carried out their tasks over a 16-week period. The findings of the study indicate that students' creative problem-posing performance differs based on the nature of the problem situation. Our findings indicate that students demonstrated better performance, particularly in the creative problem-posing context. Additionally, we found that significant and consistent relationships were observed between fluency, flexibility, and originality in students' creative problem-posing performance. The study's findings support the role of creativity components in RME for developing problem-posing competence, highlighting the need for further research in this field.

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Introduction

Although problem-posing is conceptualized from different perspectives depending on its specific purpose, the earliest discussions on this topic can be traced back to the horizon-expanding approaches to problem-solving. For instance, Pólya (1957) underscored the role of problem-posing within problem-solving in his work on heuristic reasoning, emphasizing that problem-solving processes often require the formulation and resolution of subsidiary problems. In this context, problem-posing is regarded as an indispensable component of problem-solving (Baumanns, 2023). Silver (1994) posits that problem-posing can occur at any stage of the problem-solving process—before, during, or after its execution. Furthermore, he contends that problem posing is an essential feature of problem-solving, with each posed problem contributing to the construction of a comprehensive solution. As part of the problem-posing process, pre-service mathematics teachers (PMTs) were also asked to solve the problems they created, identify potential errors, and revise their statements accordingly. The rationale behind this approach is that a key indicator of effective problem-posing is the restructuring of the problem throughout the solution process. This principle is also emphasized in the second step of Pólya's (1957) problem-solving model, which consists of understanding the problem, devising a plan, executing the plan, and assessing the results. Problem-posing activities are not necessarily integrated into the problem-solving process. Therefore, the primary objective should be to develop novel problems informed by prior experiences (Stoyanova & Ellerton, 1996). Moreover, problem-posing is acknowledged as a fundamental cognitive process in scientific research and discovery (Cai et al., 2015). Numerous studies indicate that problem-posing activities that involve problem-solving processes contribute to students' conceptual understanding (e.g., Chen et al., 2015; Silver & Cai, 1996; Singer et al., 2013).

A wide range of definitions of problem-posing can be found in the literature (e.g., Cai & Hwang, 2002; Silver, 1994; Stoyanova & Ellerton, 1996). However, in a broader sense, problem-posing can be framed as a learning strategy that involves generating new problems within a given context and restructuring them based on a particular perspective (Baumanns & Rott, 2022). Stoyanova and Ellerton (1996) define problem-posing as a process of interpreting concrete situations, stressing the importance of individual mathematical experiences. They categorize problem-posing processes into three primary types: free, semi-structured, and structured. A free problem situation entails generating problems from natural or constructed contexts; a semi-structured problem situation necessitates the examination of an open-ended scenario's structure based on prior knowledge, skills, and experiences; and a structured problem situation involves generating new problems derived from a predefined problem statement (Baumanns, 2023). Within a conceptual framework, problem posing can be designed for instructional purposes or embedded as a latent objective within open-ended or inquiry-based problem-solving environments (Cifarelli & Cai, 2005; Koichu, 2020). In this regard, problem posing serves as an open-ended cognitive process, fostering deep learning by offering opportunities for rich, content-driven tasks that enhance students' understanding (Kattou et al., 2013; Leikin & Lev, 2013).

Problem-posing activities are essential for facilitating the abstraction of real-life experiences to foster a deeper understanding of mathematics. However, mathematical problems encountered in schools often differ from and often lack alignment with real-world problems, thereby complicating the establishment of connections between

formal mathematical concepts and real-life contexts (Sulistyowati et al., 2024). In this regard, Realistic Mathematics Education (RME) offers a valuable framework for understanding how mathematics can be effectively taught and learned (Van den Heuvel-Panhuizen, 2003). As a theory guided by the principle that "mathematics is a human activity," RME comprises mathematical activities that enable students to rediscover mathematical concepts (Freudenthal, 1991). Mathematical activity involves the mathematization of problem situations originating from mathematical structures or real-life contexts (Gravemeijer, 1994). According to Freudenthal (1991), problem-posing situations represent a creative process that incorporates real-life contexts and human interactions to foster rich and meaningful mathematical experiences. Building upon this perspective, we designed the creative problem-posing situations in our study to highlight their integration with real-life contexts. This approach is based on the idea that one of the most effective ways to address real-world problems involves structuring problem situations within realistic contexts.

Furthermore, we incorporated creativity—an aspect often overlooked in educational research until recently—into our study to foster a comprehensive understanding of PMTs problem-posing behaviors. Given the potential of problem-posing to influence mathematics teaching and learning, it is evident that it plays an essential role in achieving the objectives of school mathematics. Accordingly, to develop a deep and effective understanding of mathematics, PMTs must acquire both knowledge and skills in problem-posing. This study introduces an instructional design aimed at enhancing PMTs' creative problem-posing skills within the framework of RME. The effectiveness of this design was examined through the following research questions (RQs):

- RQ1. To what extent did PMTs achieve creative problem-posing proficiency in various problem-posing contexts?
- RQ2. To what extent did PMTs exhibit creative problem-posing proficiency across the three dimensions of creativity—fluency, flexibility, and originality?
- RQ3. Is there a significant relationship between PMTs' problem-posing proficiency and the three dimensions of creativity? If so, how strong is this relationship?

Theoretical Background

Problem-Posing Skills

One of the central research foci in problem-posing lies in the relationship between problem-posing and problem-solving (e.g., Cai & Hwang, 2002; Kilpatrick, 1987; Silver & Cai, 1996). Consequently, a significant portion of the existing literature has predominantly focused on problem-solving. However, researchers emphasize that developing the ability to formulate mathematical problems is equally important as improving problem-solving skills (Stoyanova & Ellerton, 1996). In this regard, Kilpatrick (1987, p. 123) asserted that "students should have the experience of discovering and creating their own mathematical problems as an integral part of their education". Curriculum guidelines also highlight the necessity of providing students with regular opportunities to design novel problems tailored to diverse contexts (e.g., Ministry of Education [MoE], 2015; National Council of Teachers of Mathematics [NCTM], 2014). If we aim to develop educators as autonomous problem solvers and problem posers, they must receive sufficient professional training (Kilpatrick, 1987; Silver & Cai, 2005). However, research indicates that many pre-service and in-service teachers lack the knowledge and expertise to extend beyond

problem-solving skills (Cai & Hwang, 2002; Crespo & Sinclair, 2008; Ran et al., 2025). Discussions on problem-posing emphasize several fundamental aspects, including the exploration of mathematical and non-mathematical contexts, the reformulation of problems, the enhancement of problem realism, the identification of alternative formulations that lead to diverse mathematical models, and the construction of accessible problems (e.g., English, 1997; Kilpatrick, 1987; Silver, 1994). A well-constructed problem should fulfill three fundamental competency criteria: (i) fostering reasoning and incorporating significant mathematical concepts (epistemic), (ii) aligning with realistic, imaginable, and relatable contexts (affective), and (iii) demonstrating coherence with the mathematics curriculum while being appropriate for the student's proficiency level (pedagogical) (Goldin, 2014; Koichu, 2020; Silver, 1997; Van den Heuvel-Panhuizen & Drijvers, 2020). This study assessed the problem-posing skills of PMTs across three key dimensions: the appropriateness of the mathematical structure (epistemic), the inclusion of realistic conceptual and contextual characteristics (affective), and alignment with the curriculum (pedagogical). In particular, factors such as data quality, resolvability and accessibility, problem complexity, and instructional integration were identified as core analytical criteria.

Creativity and Its Relationship with Problem-Posing

Creativity, as a cognitive ability, is directly associated with mathematical problem-posing processes (Bonotto & Santos, 2015). In essence, problem-posing is the structured manifestation of mathematical creativity, necessitating structured process organization within specific contexts. Within problem-posing processes, creative teaching competencies—such as fluency, flexibility, and originality—become evident through the interplay between subject knowledge and contextual structure. Decades ago, Kilpatrick (1987) posed the question, ‘Where do good problems come from?’ without anticipating its profound impact on subsequent research. In this study, we have purposefully integrated creativity with problem-posing in a more concrete manner, embedding it into real-world contexts. This approach is based on the idea that one of the most fundamental aspects—arguably the most significant—of problem-posing is creativity. Leikin (2009) describes creativity as a dynamic trait that should be conceptualized as a "habit of mind". Therefore, within the scope of problem-posing research, creativity functions as a fundamental indicator in assessing learning outcomes.

Research highlights a strong correlation between problem-posing skills and the fundamental components of creativity—flexibility, fluency, and originality (Cai & Hwang, 2002; Kontorovich et al., 2011; Leikin & Sriraman, 2022; Silver, 1997; Silver & Cai, 1996). Consequently, problem finding sensitivity and the importance of problem-solving are considered integral to defining the creative process (Bonotto & Santo, 2015). In this context, problem-posing activities are employed to both foster and assess creativity (e.g., Leikin, 2009; Torrance, 1966). Existing studies primarily examine creativity across the dimensions of flexibility, fluency, and originality. For instance, Kontorovich et al. (2011) assessed students' creativity in problem-posing using these three dimensions. This approach aligns with Torrance's (1962/1974) framework, which links creative thinking to problem-solving abilities. However, there is no consensus among researchers regarding the nature of this relationship (Bonotto & Santo, 2015; Yuan & Sriraman, 2011). This study aims to develop a method for analyzing the relationship between problem-posing activities, grounded in RME, and the components of creativity. We propose a roadmap for identifying and evaluating PMTs' creativity in problem-posing processes. Furthermore, we argue that the

fundamental dimensions of creativity—fluency, flexibility, and originality—can be enhanced, and that PMTs' competencies in these areas can be further cultivated.

Measuring Problem-Posing in Terms of Creativity

The fundamental characteristic of creative thinking is the ability to develop diverse and original solutions to problems (Silver, 1997). Cai and Leikin (2020) argue that problem-posing can serve as a tool in achieving instructional objectives, such as fostering mathematical creativity and enhancing competencies. In this context, ensuring the measurability of creativity is essential. In our study, we focused on three key dimensions of creativity: fluency, flexibility, and originality. One of the core theoretical foundations in creativity research is Guilford's (1950) distinction between divergent and convergent thinking. Guilford conceptualized creative thinking as a category of productive cognition and defined nine essential cognitive skills: problem sensitivity, synthesis ability, cognitive flexibility, fluency, originality, analytical reasoning, restructuring ability, ideational scope, and evaluative skills. Chapman (2012) emphasizes that problem-posing ability is closely linked to mathematical knowledge, creativity, and problem-solving experience. Consequently, different approaches exist for assessing creativity in problem-posing contexts. For instance, Tabach and Friedlander (2015) evaluated students' problem-posing activities to assess group performance. They measured flexibility based on the diversity of problem types, fluency by the number of generated problems, and originality using the mean number of unconventional modifications. Similarly, Leikin and Elgrably (2020) examined problem-posing through research-oriented activities to assess PMTs' creative performance. They evaluated fluency by the number of discovered features, flexibility by the variety of problem types addressed, and originality based on the novelty and infrequency of the examined features. In most studies, fluency, flexibility, and originality are regarded as key indicators of creativity assessment (e.g., Chapman, 2012; Leikin & Elgrably, 2020; Tabach & Friedlander, 2015). However, nuances in the relationship between problem-posing and creativity have been emphasized, particularly the impact of experience and preparation (Voica & Singer, 2013), in addition to aesthetic criteria (Crespo & Sinclair, 2008). Within this framework, fluency, flexibility, and originality serve as fundamental criteria for evaluating creativity. This study adopts the creativity attributes proposed by Guilford (1950), along with fluency, flexibility, and originality—dimensions widely applied in Torrance's tests and numerous other studies (Cai et al., 2015; Kontorovich et al., 2011; Silver, 1997).

Methodology

Study Design and Participants

This study is framed as a problem-posing activity enriched with real-world contexts, incorporating the development, refinement, and enhancement of mathematical problems. Accordingly, it focuses on the three core dimensions of creativity—fluency, flexibility, and originality—which are recognized as essential components of problem-posing proficiency and its developmental stages. The research follows a mixed-methods design, incorporating both qualitative and quantitative dimensions. The qualitative aspect examines participants' problem-posing processes, while the quantitative aspect evaluates the fluency, flexibility, and originality scores of the generated problems. The study participants comprise 40 PMTs enrolled at a public university. Participants were

selected using a criterion-based sampling method. The first criterion required successful completion of the Mathematics Learning and Teaching Approaches course, a mandatory component of the teacher education program in Turkey. This requirement ensured that participants had foundational knowledge of the RME approach, its philosophical underpinnings, design principles, horizontal and vertical mathematization, and classroom applications. The second criterion was enrollment in the Problem-Solving in Mathematics Teaching course, taught by the researcher. This course covers key concepts such as problem-posing, problem-solving, problem-solving and problem-posing strategies, classification of problem-posing, and the role of problem posing and solving in the curriculum (Council of Higher Education [CoHE], 2024). Among the participants, 70% ($n=28$) were female, and 30% ($n=12$) were male. The study was conducted during the Fall semester of the 2023-2024 academic year.

Design and Conduct of the Study

As part of the design-based research, an instructional program was first developed, aligned with the course syllabus. Accordingly, the content of the Problem Solving in Mathematics Teaching education course was expanded to incorporate RME, and creativity. The course was structured to span 16 weeks. During the first eight weeks, integrated activities were implemented to support creative problem-posing, embedded within the framework of RME. The following three weeks focused on instructional design practices. The first of the final five weeks centered on theoretical discussions and practical applications related to the overall course structure, while the remaining four weeks were dedicated to individual research and projects.

Table 1. Integrated Course Plan Designed for PMTs

Integrated course	Week	Duration	Course activity	Description of the course activity	
Creative problem posing-solving in the context of RME	1 st week	30 min	Warming up [CCD]	Thinking about what posing/solving problems means as a concept and sharing this with a partner	
		30 min	Small group work [CCD]	Sharing initial thoughts about the importance of problem situations in the mathematics curriculum	
		30 min	Whole group sharing [CCD & RCD]	Achieving consensus on the definition, process, characteristics and importance of problem posing in mathematics teaching	
	2 nd week	45 min	Small group elaboration [CCD]	Elaboration and discussion of mathematical problem solving strategies	
		45 min	Whole group synectic [RCD]	Combining the problem-solving process and discussing its implications for mathematics teaching	
	3 rd week	90 min	Whole group brainstorming [RCD]	Raising awareness of problem-posing processes and organizing their reflections on teaching	
	4 th week	45 min	Small group work [CCD]	The first attempts at alternative evaluation systems through problem posing/solving	
		45 min	Small group work [CCD]	The second attempt on course designs for different learning areas with problem posing/solving	
		30 min	Whole group synectic [CCD & RCD]	The concept of creativity in all its aspects and evaluation of the necessity of creativity in terms of mathematics teaching	
		5 th week	30 min	Whole group brainstorming	Identifying situations that characterize creativity and discussing possible creativity indicators

Integrated course	Week	Duration	Course activity	Description of the course activity
			[RCD]	
		30 min	Small group talking ring	Discovering the features needed for creative mathematics education to produce useful results
	6 th week	45 min	Whole group sharing	Creative problem characteristics, evaluation and discussion of the relationship between problem creativity and problem posing
		45 min	Whole group elaboration	Contextual elaboration of the relationship between creativity as a process and problem posing/solving
	7 th week	90 min	Small group work	Philosophical aspect of writing creative situations and creative problems in RME
		45 min	Small group synectic	The first attempt to create creative problems and creative designs in RME
	8 th week	45 min	Small group synectic	The second attempt to create creative problem posing exercises in the context of RME
	9 th week	90 min	Small group work	Creative structured problem-posing instructional design and experience in the context of RME
Instructional design experience	10 th week	90 min	Small group work	Creative semi-structured problem-posing instructional design and experience in the context of RME
	11 th week	90 min	Small group work	Creative free problem posing instructional design and experience in the context of RME
General procedural discourses	12 th week	90 min	Focus group interviews	Reflecting on creative problem posing/solving procedures, sharing the process, and evaluating group reflections in the context of RME
Individual study procedure	13 th week	90 min	Individual and tutorial support [RID]	Individually experiencing creative structured problem-posing instructional design and experience in the context of RME
	14 th week	90 min	Individual and tutorial support [RID]	Individually experiencing creative semi-structured problem-posing instructional design and experience in the context of RME
	15 th week	90 min	Individual and tutorial support [RID]	Individually experiencing creative free problem-posing instructional design and experience in the context of RME
Final procedure discourses	16 th week	90 min	Focus tutorial discussions [RID]	Thinking about creative problem posing/solving procedures in the context of RME, sharing the process and evaluating its reflections as an instructor.

[CCD]: Communal Cluster Discourses [RCD]: Reflective Cluster Discourses [RID]: Reflective Individual Discourses

The theoretical foundation of this process is based on Shulman and Shulman's (2004) fostering a community of learners (FCoL), Schön's (1983) concept of reflective practitioners, and Vygotsky's (1934/1986) social development theory, emphasizing reflective and collaborative group structures. The processes outlined in Table 1 integrate theoretical knowledge on mathematically rich and contextually creative real-world problems. The structure includes small and large group discussions, collaborative learning, brainstorming, talking circles, synectics, focus group interviews, and guided instructional discussions. The reflections, contributions, and evaluations of participants in group work and focus group discussions constitute the reflective learning community. Individual studies, represent the original designs authored by participants based on their group experiences. As part of the course, PMTs completed structured, semi-structured, and free problem-posing tasks and subsequently solved the problems they created. They were required to provide detailed explanations of all possible solution processes. In the problem-posing tasks, participants were given the flexibility to determine the

and data quality, contextuality and reality, complexity and directive clarity, and fitness for purpose. This evaluation was conducted at three proficiency levels: high, moderate, and low. Each category included predefined criteria corresponding to structured, semi-structured, and free problem types. Three criteria were set for high and moderate levels, while four criteria were defined for the low level. According to the evaluation criteria: A total score of 7 or higher in a category was classified as high-level proficiency. A score between 4 and 6 (inclusive) was classified as moderate-level proficiency. A score below 4 was classified as low-level proficiency. This classification system aimed to provide a more detailed assessment of problem-posing performance. The problem-posing assessment tool was adapted from existing evaluation frameworks and criteria in the literature (e.g., Bonotto & Santo, 2015; Cai & Hwang, 2002; Cifarelli & Cai, 2005; Chen et al., 2015; Crespo & Sinclair, 2008; Koichu, 2020; Ozgen, 2023; Silver, 1994; Silver & Cai, 2005; Stoyanova & Ellerton, 1996). Detailed information regarding the assessment process is presented in Table 3.

Table 3. Problem-Posing Evaluation Table for PMTs

Evaluation Categories	Descriptions Levels		
	High #	Medium #	Low #
Mathematicality Structure	<ul style="list-style-type: none"> ▪ The transfer between the mathematical text and all relevant contextual factors are planned correctly [3*]. ▪ The required structures such as mathematical concepts, symbols, representations, units, expressions and visuals (figures, tables, graphs, <i>etc.</i>) have been constructed accurately and completely [3*]. ▪ The starting and ending points of the mathematical problem (creating the problem that preserves the mathematical structure) are clear and also reflect the mathematical language [3*]. 	<ul style="list-style-type: none"> ▪ There are deficiencies in the transfer between the mathematical text and all associated contextual factors. ▪ There are deficiencies in required structures such as mathematical concepts, symbols, representations, units, expressions and visuals (figures, tables, graphs, <i>etc.</i>). ▪ Even though there are structural problems in the relationship between the starting and ending point of the mathematical problem (creating the problem that preserves the mathematical structure), it reflects the mathematical language. 	<ul style="list-style-type: none"> ▪ No information is available, the information given in the problem is meaningless, there is no fit between the text and the associated contextual factors. ▪ The transfer between the mathematical text and all associated contextual factors is not planned correctly. ▪ Necessary structures such as mathematical concepts, symbols, representations, units, expressions and visuals (figures, tables, graphs, <i>etc.</i>) were not used appropriately. ▪ The starting and ending point of the mathematical problem is unclear and no fluency in the mathematical language.
Solvability & Data Quality	<ul style="list-style-type: none"> ▪ The solution of the problem includes situations that are reasonable, logical, realistic and clearly express the desired unit (providing access to the intended mathematical results) [3*]. ▪ The information given for the solution is accurate, error-free and allows multiple solutions [3*]. ▪ The data related to the problem are mathematically and logically compatible, contribute to the meaningfulness of the result and qualify the existence of the unit [3*]. 	<ul style="list-style-type: none"> ▪ The solution to the problem is reasonable, logical, realistic, but contains uncertainties in clearly expressing the desired unit (providing access to the intended mathematical results). ▪ The information provided for the solution is accurate and error-free, but it has limitations in allowing multiple solutions. ▪ Although the data related to the problem are not in full mathematical and logical harmony, they contribute to the meaningfulness of 	<ul style="list-style-type: none"> ▪ There is no information available, the problem solution is meaningless, there is no match between the solution and the data quality. ▪ The solution to the problem is not sufficient to achieve the intended mathematical results. ▪ The information given for the solution does not match the problem situation and does not allow multiple solutions. ▪ The data related to the problem contains mathematical and logical errors and the meaningfulness of the result is uncertain.

Evaluation Categories	Descriptions Levels		
	High #	Medium #	Low #
	the result.		
Contextuality & Reality	<ul style="list-style-type: none"> ▪ The problem is associated with daily life (high probability of occurrence is emphasized) and includes situations using realistic data [3*]. ▪ Judgments regarding a solution and an answer that would be accepted in a real-life situation are consistent [3*]. ▪ Life science supports solutions and hypothesis [3*]. 	<ul style="list-style-type: none"> ▪ The problem is associated with daily life (emphasis is placed on the high probability of its occurrence) but there are problems in using realistic data. ▪ There are inconsistencies in judgments regarding the answer and producing a solution that will be accepted in real life situations. ▪ Life science does not fully support solutions and assumptions. 	<ul style="list-style-type: none"> ▪ No information is available, no real-world context has been established for the problem, and there is no coherence between the text and associated contexts. ▪ The problem is associated with the context of daily life (emphasis is placed on the high probability of occurrence), but the data quality is not consistent with the problem situation. ▪ There is no acceptable solution in a real-life situation. ▪ Life science does not support solutions and assumptions.
Complexity & Directive	<ul style="list-style-type: none"> ▪ The problem involves the cognitive effort that students must make [3*]. ▪ It requires engaging in reasoning, planning, analysis, judgment, and critical and creative thinking. It has instructions that require abstract and complex thinking [3*]. ▪ It contributes to operational fluency and has high transfer power to mathematical and daily language (terms, expressions, operations, etc.) [3*]. 	<ul style="list-style-type: none"> ▪ The problem involves the cognitive effort that students must make, but does not fully reflect engaging in reasoning, planning, analysis, judgment, and critical and creative thinking. ▪ There are deficiencies in instructions that require abstract and complex thinking. ▪ It contributes to operational fluency, but its transfer power into mathematical and daily language is not sufficient. 	<ul style="list-style-type: none"> ▪ No information is available; the problem situation does not provide cognitive and operational complexity. ▪ The problem does not fully reflect the cognitive effort that students must put forth. ▪ They do not have situations that require reasoning, planning, analysis, judgment, and engaging in critical and creative thinking. It does not contain instructions that require abstract and complex thinking. ▪ The transfer of procedural fluency into mathematical and daily language is weak.
Fitness for Purpose	<ul style="list-style-type: none"> ▪ The problem purpose reflects the curriculum outcome(s) [3*]. ▪ The problem situation is plausible and requires conscious use [3*]. ▪ The purpose of the problem serves the selection of the appropriate solution and the use of tools [3*]. 	<ul style="list-style-type: none"> ▪ The problem purpose includes the curriculum outcome(s). ▪ There are uncertainties in the plausibility and conscious use of the problem situation. ▪ The purpose of the problem is not sufficient to serve the appropriate solution selection and tool use. 	<ul style="list-style-type: none"> ▪ No information is available; the problem situation does not contain a purpose. ▪ The problem purpose partially reflects the curriculum outcome(s). ▪ There are many limitations in the plausibility and conscious use of the problem situation. ▪ It is insufficient to serve the purpose of the problem, appropriate solution selection and tool use.
Criteria	0 1 2 3 4 5 6 7 8 9	0 1 2 3 4 5 6 7 8 9	0 1 2 3 4 5 6 7 8 9
Indicator Score	Criterion indicator score ≥ 7	4 ≤ Criterion indicator score ≤ 6	Criterion indicator score ≤ 3

◆It represents the maximum score that can be obtained from the relevant criterion indicator.

In the second stage of coding, students' problem-posing competencies were independently analyzed in relation to

the core indicators of creativity: fluency, flexibility, and originality. When evaluating solution processes aligned with creativity indicators, definitions and explanations from the relevant literature were referenced (Guilford, 1950; Kattou et al., 2013; Klavir & Gorodetsky, 2011; Leikin & Lev, 2013; Lu & Kaiser, 2022; Mann, 2006; Silver, 1997; Sriraman, 2009; Sternberg & Lubart, 1999). Torrance (1962/1974) proposed a framework for creative thinking related to problem-solving abilities, identifying three key dimensions: fluency, flexibility, and originality. He defined fluency as the ability to generate ideas continuously, flexibility as the capacity to approach a problem from multiple perspectives and produce diverse solutions, and originality as the ability to generate novel and unconventional solutions. The creativity indicators adapted for this study were refined based on structured, semi-structured, and free problem-posing contexts, as shown in Table 4.

Table 4. Proficiency Demonstrations of Mathematical Problem-Posing

Creativity Indicators	Descriptions of problem posing levels in the context of their structural features		
	High #	Medium #	Low #
Fluency	<p>▲The mathematical structure of the problem encourages the construction of a specific solution path, containing effective concept(s) and procedures for understanding solution structures and exploring possible relationships between problem statements and solution ideas.</p> <p>▼The problem situation allows effective use of alternative solutions by using the knowledge, skills, concepts and relationships in mathematical experiences.</p> <p>►Problem situations where knowledge is reflected without any boundaries or restrictions help to produce multiple solutions and use effective mathematical communication tools in an organized way.</p>	<p>▲Although the mathematical structure of the problem contains qualified data for the development of solutions, it contains deficiencies in terms of both contextuality and the generation of solution ideas.</p> <p>▼Although the problem situation encourages the use of alternative solutions by using the knowledge, skills, concepts and relationships in mathematical experiences, it has limitations.</p> <p>►Although problem situations in which information is reflected without any boundaries or restrictions encourage the generation of multiple solutions, they have limitations in terms of data quality.</p>	<p>▲The mathematical structure of the problem is not of a quality that would encourage the development of special solutions and ideas.</p> <p>▼The problem situation is not capable of revealing and applying the knowledge, skills, concepts and relationships in mathematical experiences.</p> <p>►Problem situations where knowledge is reflected without any boundaries or restrictions do not contain suitable content for multiple solutions, the use of mathematical language, and generating alternative ideas.</p>
	Flexibility	<p>▲The mathematical structure of the problem provides an effective content for asking special ideas, answers and questions, looking at the problem situation from a different perspective, and developing an approach and way of thinking.</p> <p>▼It helps to effectively use different representation systems to help find alternatives and directions for the problem situation, and to establish clear and consistent relationships between data and objectives.</p> <p>►Problem situations in which</p>	<p>▲Although the mathematical structure of the problem contains structural content for asking special ideas, answers and questions, it contains limitations for looking at the problem situation from a different perspective and developing an approach and way of thinking.</p> <p>▼Although the problem situation allows the effective use of different representation systems to help find alternatives and directions, it has limitations in establishing clear and consistent relationships between data and objectives.</p>

Creativity			
Descriptions of problem posing levels in the context of their structural features			
Indicators	High #	Medium #	Low #
Originality	knowledge is reflected without any boundaries or restrictions allow using effective strategies and making appropriate representation choices by considering different possibilities.	▶Although problem situations in which information is reflected without any restrictions are created in a context where different possibilities are taken into account, they contain limitations regarding the choice of strategy and appropriate representation.	relationships. ▶Problem situations in which information is reflected without any boundaries or restrictions do not take into account different possibilities and do not allow the use of appropriate representation choices.
	▲The mathematical structure of the problem enables the emergence of new and unique ideas, providing innovative content for the effective use of unusual shapes, diagrams, tables and representations. ▼It offers innovative perspectives on the effective use of extraordinary strategies by allowing the creation of part(s), elements and unusual combinations related to the problem situation. ▶Problem situations in which knowledge is reflected without any boundaries and restrictions provide a source for thinking and interpreting unusual solutions, allowing to produce innovative designs and develop them effectively.	▲The mathematical structure of the problem allows for the emergence of new and unique ideas and includes fictional problems for the effective use of unusual shapes, diagrams, tables and representations. ▼The possibility of creating part(s), elements and unusual combinations related to the problem situation also has limitations in the use of unusual strategies. ▶Although problem situations where information is reflected without any boundaries and restrictions provide a source for thinking and interpreting unusual solutions, they do not contain data of the quality and quality that can produce innovative designs.	▲The mathematical structure of the problem was not designed to the quality and quality that would enable new and unique ideas to emerge. ▼It does not allow creating part(s), elements and unusual combinations related to the problem situation. ▶Problem situations where knowledge is reflected without any boundaries or restrictions are not of a quality that will allow the production of innovative designs by creating a source for thinking and interpreting unusual solutions.

▲: Structured ▼: Semi-Structured ▶: Free

Data Analysis

Based on students' responses, the coding of problem-posing proficiency and creativity indicators was conducted independently by the first author and a field expert who teaches the problem-posing in mathematics education course at the undergraduate level. To ensure the reliability of the coding process, the responses of all 40 students were analyzed. The inter-rater agreement was measured using Cohen's kappa coefficient, yielding a value of 0.84. According to Altman (1991), a weighted kappa value of ≥ 0.81 across all dimensions indicates a "very good" or nearly perfect agreement (p. 404).

For each problem-posing task (structured, semi-structured, and free), problem-posing proficiency indicators and the three dimensions of creativity were first analyzed through descriptive statistics (frequency and percentage). Subsequently, Friedman's test was applied to compare students' proficiency and creativity levels across problem-posing tasks. This test serves as the non-parametric equivalent of one-way repeated measures ANOVA. To identify the source of significant differences among indicators, Dunn-Bonferroni post hoc tests were applied. Finally, Spearman's rank correlation was applied to assess students' performance correlations across different indicators, while partial correlation analysis was conducted to examine how problem-posing proficiency relates

to the three dimensions of creativity. These analyses provided a comprehensive understanding of how different task structures influence the relationship between problem-posing skills and creativity.

Results

PMTs' Performance Based on Their Problem-Posing Proficiency (RQ1)

Table 5. PMTs' Proficiency Levels by Problem Situations: Percentages and Frequencies

Levels	Structured	Semi-structured	Free
Low	7.5% [3]	12.5% [5]	7.5% [3]
Medium	30.0% [12]	37.5% [15]	12.5% [5]
High	62.5% [25]	50.0% [20]	80.0% [32]

Note: Each PMTs designed activities appropriate to structured, semi-structured and free problem situations. Depending on the number of participants, 120 (3*40) structured, semi-structured and free problem posing situations were evaluated.

As indicated in Table 5, 80% of PMTs demonstrated successful approaches and effective initiatives in free problem-posing tasks, while 50% performed similarly in semi-structured tasks and 62.5% in structured tasks. Notably, the proportion of students with low proficiency across all three problem-posing contexts was minimal. According to the Friedman test, significant differences were observed in PMTs' problem-posing proficiency across the three problem-posing conditions ($\chi^2(2)=15.474, p<0.001$). Following the Bonferroni correction, results from the Dunn-Bonferroni post hoc test revealed statistically significant differences between structured and semi-structured ($p<0.05$), structured and free ($p<0.05$), and semi-structured and free ($p<0.001$) problem-posing conditions. These findings indicate that students achieved the highest performance in free problem-posing tasks and the lowest in semi-structured problem-posing tasks. In particular, PMTs formulated higher-quality problems in free tasks across key dimensions, including mathematical structure, solvability, data quality, realism, complexity, directive formulation, and alignment with the curriculum. This suggests that when PMTs are aware of the problem-posing process, recognize the importance of problem-solving, and are encouraged to develop independent ideas, they tend to perform at higher levels.

PMTs' Performance in Terms of Creativity (RQ2)

PMTs' proficiency in problem-posing tasks was analyzed based on the key dimensions of creativity: fluency, flexibility, and originality. The level of creativity in their problem-posing approaches was evaluated based on criteria such as the appropriateness of the methods chosen for the given problem, the validity of unconventional solution paths, and the quality of ideas generated through divergent, fluent, and flexible thinking.

Performance of PMTs' According to Fluency Indicator

Table 6 indicates that 57.5% of PMTs exhibited a high level of fluency performance in structured problem-posing tasks, while 37.5% did so in semi-structured tasks and 77.5% in free tasks. Additionally, 32.5% of participants demonstrated a moderate level of fluency in structured tasks, 47.5% in semi-structured tasks, and 17.5% in free

tasks. Significant differences in fluency performance were observed across all three problem-posing conditions ($\chi^2(2)=17.575$, $p<0.001$). Post hoc analyses revealed statistically significant differences among problem-posing conditions, specifically between structured and semi-structured ($p<0.05$), structured and free ($p<0.05$), and semi-structured and free ($p<0.001$). PMTs demonstrated the highest level of fluency in free problem-posing tasks, whereas the lowest fluency was recorded in semi-structured tasks. These findings suggest that greater awareness of the ongoing nature of problem-posing and idea continuity could have further enhanced participants' fluency.

Table 6. PMTs' Fluency Levels in the Problem-Posing Task: Percentages and Frequencies

Levels	Structured	Semi-structured	Free
Low	10.0% [4]	15.0% [6]	5.0% [2]
Medium	32.5% [13]	47.5% [19]	17.5% [7]
High	57.5% [23]	37.5% [15]	77.5% [31]

Performance of PMTs' According to Flexibility Indicator

Table 7 shows that 60% of PMTs exhibited a high level of performance in flexibility in structured problem-posing tasks, while 27.5% did so in semi-structured tasks and 75% in free tasks. Additionally, 25% of participants demonstrated a moderate level of flexibility in structured tasks, 50% in semi-structured tasks, and 22.5% in free tasks. Significant differences in performance in flexibility were observed across the three problem-posing conditions ($\chi^2(2)=23.281$, $p<0.001$). Post hoc analyses revealed statistically significant differences among problem-posing conditions, specifically between structured and semi-structured ($p<0.05$), structured and free ($p<0.05$), and semi-structured and free ($p<0.001$). PMTs demonstrated the highest level of flexibility in free problem-posing tasks, whereas the lowest level was recorded in semi-structured tasks. This outcome can be attributed to their ability to generate a wider range of alternative solutions. In contrast, in semi-structured problem-posing tasks, participants struggled to recognize the potential for multiple outcomes and encountered difficulties in developing diverse solution strategies.

Table 7. PMTs' Flexibility Levels in the Problem-Posing Task: Percentages and Frequencies

Levels	Structured	Semi-structured	Free
Low	15.0% [6]	22.5% [9]	2.5% [1]
Medium	25.0% [10]	50.0% [20]	22.5% [9]
High	60.0% [24]	27.5% [11]	75.0% [30]

Performance of PMTs' According to Originality Indicator

Table 8 indicates that 55% of PMTs exhibited a high level of performance in originality in structured problem-posing tasks, while 60% did so in semi-structured tasks and 77.5% in free tasks. Additionally, 7.5% of participants demonstrated a moderate level of originality in structured tasks, 10% in semi-structured tasks, and 5% in free tasks. Significant differences in originality were found among the three problem-posing conditions ($\chi^2(2)=6.636$, $p<0.05$). Post hoc analyses revealed statistically significant differences among problem-posing conditions,

specifically between structured and free ($p<0.05$) as well as semi-structured and free ($p<0.05$). However, no significant difference was found between structured and semi-structured conditions ($p>0.05$). PMTs demonstrated the highest level of originality in free problem-posing tasks. This outcome suggests that more diverse problem selection supported originality, while the lower originality in structured tasks reflects their restrictive nature.

Table 8. PMTs' Originality Levels in the Problem-Posing Task: Percentages and Frequencies

Levels	Structured	Semi-structured	Free
Low	7.5% [3]	10.0% [4]	5.0% [2]
Medium	37.5% [15]	30.0% [12]	17.5% [7]
High	55.0% [22]	60.0% [24]	77.5% [31]

Correlations between Proficiency in Problem-Posing and Creativity (RQ3)

The problem-posing tasks of PMTs incorporate features that foster creativity. In this study, we analyzed the relationships among the observed components of creativity in students' problem-posing performance. This analysis aimed to provide a broader perspective on the relationship between the established problem-posing criteria and these three dimensions of creativity. Spearman correlation was used to examine the relationship between mathematical problem-posing proficiency and creativity indicators, providing deeper insights. Table 9 indicates that the correlations between problem-posing proficiency and the components of creativity are statistically significant.

Table 9. Relationships between Different Aspects of PMTs' Performance on Problem-Posing Tasks

Tasks	Creativity Indicator	Proficiency	Fluency	Flexibility	Originality
Structured	Proficiency	1	0.624**	0.702**	0.536**
	Fluency		1	0.426**	0.530**
	Flexibility			1	0.452**
	Originality				1
Semi-structured	Proficiency	1	0.711**	0.573**	0.776**
	Fluency		1	0.440**	0.568**
	Flexibility			1	0.528**
	Originality				1
Free	Proficiency	1	0.694**	0.591**	0.694**
	Fluency		1	0.364*	0.582**
	Flexibility			1	0.730**
	Originality				1

* $p<0.05$, ** $p<0.01$

The correlation coefficients between proficiency and fluency were calculated as follows: $r_s(40)=0.624$, $p<0.01$ for structured problem-posing, $r_s(40)=0.711$, $p<0.01$ for semi-structured problem-posing, and $r_s(40)=0.694$, $p<0.01$ for free problem-posing. These findings suggest a strong and significant relationship between proficiency and fluency. The correlation coefficients between proficiency and flexibility were $r_s(40)=0.702$, $p<0.01$ for structured,

$r_s(40)=0.573, p<0.01$ for semi-structured, and $r_s(40)=0.591, p<0.01$ for free problem-posing. These results indicate that the relationship between proficiency and flexibility ranges from moderate to strong. The correlation coefficients between proficiency and originality were $r_s(40)=0.536, p<0.01$ for structured, $r_s(40)=0.776, p<0.01$ for semi-structured, and $r_s(40)=0.694, p<0.01$ for free problem-posing. These findings suggest that the relationship between proficiency and originality varies from moderate to strong.

Table 10. Partial Correlations between Different Aspects of PMTs' Performance on Problem-Posing Tasks

Tasks	Creativity Indicator	Fluency	Flexibility	Originality
Structured	Fluency	1	-0.022	0.295
	Flexibility		1	0.125
	Originality			1
Semi-structured	Fluency	1	0.057	0.037
	Flexibility		1	0.162
	Originality			1
Free	Fluency	1	-0.079	0.195
	Flexibility		1	0.550**
	Originality			1

** $p<0.01$

To exclude the influence of problem-posing proficiency on the measurement of creativity, we re-examined the correlations among creativity dimensions while controlling for the proficiency variable. Table 10 indicates that, when controlling for proficiency in structured and semi-structured problem-posing contexts, no significant differences were found among the dimensions of creativity (fluency, flexibility, originality). This result suggests that structured and semi-structured problem-posing proficiency has a significant impact on these correlations. Similar findings were obtained for free problem-posing; however, this was not the case for the correlation between flexibility and originality.

Discussion

This study focused on measuring the dimensions of fluency, flexibility, and originality in problem-posing tasks within the framework of RME, which requires creative thinking. In this context, we first emphasized the complex relationship between creativity and problem-posing proficiency. Numerous studies conducted in different countries and at various educational levels have explored problem-posing proficiency and examined the effects of creativity (e.g., Bonotto & Santo, 2015; Kattou et al., 2013; Mann, 2006; Silver, 1997; Singer et al., 2013; Yuan & Sriraman, 2011). For instance, Cai and Hwang (2002) examined the relationship between problem-posing and problem-solving among American and Chinese sixth-grade students, using answer accuracy, solution strategy type, and solution representation as assessment criteria. Their findings revealed a strong connection between problem-posing and problem-solving among Chinese students, whereas this relationship was weaker among American students. Kontorovich et al. (2011) proposed that problem generation consists of knowledge base, heuristic methods, individual assessments, group dynamics, and task organization, emphasizing the strengths of

creativity in the evaluation process. Similarly, we examined PMTs' problem-posing processes and observed that students actively attempted to demonstrate the strengths of creativity. Thus, adopting a RME perspective, we found that fluency, flexibility, and originality in problem-posing contexts play a crucial role in fostering creativity.

In problem-posing proficiency, the dimensions of fluency, flexibility, and originality are recognized as key indicators of creativity (Kontorovich et al., 2011). Our findings indicate that problem-posing proficiency and creativity can be assessed through similar tasks, and these skills are expressed most strongly in free problem-posing contexts. PMTs performed worse in semi-structured problem-posing tasks than in other problem-posing formats. This may be explained by their limited mathematical experience. Stoyanova and Ellerton (1996) also emphasize that semi-structured problem-posing requires mathematical experience and is designed to facilitate the exploration of knowledge, skills, concepts, and relationships. In our study, we observe that supporting PMTs' structured, semi-structured, and free problem-posing skills enables most of them to reach moderate to high proficiency levels. The fact that only a small number of students exhibited low proficiency highlights the potential of enriched instructional plans in enhancing problem-posing skills. Accordingly, in our study, PMTs were asked to solve and analyze their own problems, which contributed to improvements in their problem-posing proficiency. Kilpatrick (1987) emphasizes the importance of students discovering their own problems, while Silver (1997) argues that for educators to become effective problem solvers and problem posers, they must be given the opportunity to analyze their own problems in depth. This approach aligns with findings from numerous studies that emphasize the strong relationship between problem-solving and problem-posing (Cai et al., 2015; Silver, 1997; Stoyanova & Ellerton, 1996). To address this, we adopted an instructional approach that integrated a well-structured course plan, a strong theoretical foundation, effective content examples, and opportunities for experiential learning to develop both problem-posing and problem-solving behaviors among PMTs. The results indicate that this process leads to significant improvements in their problem-posing proficiency.

Based on PMTs' problem-posing practices, we examined how problem-posing influences creativity competencies throughout the process. Accordingly, we sought to provide high-quality findings to the field by referencing recent studies on problem-posing and its creative components (e.g., Bonotto & Santo, 2015; Cai & Hwang, 2002; Leikin, 2009; Leikin & Sriraman, 2022; Lu & Kaiser, 2022). Our findings suggest that PMTs exhibited strong fluency, flexibility, and originality, particularly in free problem-posing tasks. We attribute this to the free nature of idea generation, which facilitates problem-posing behaviors and promotes the development of more innovative problem content. While examining the relationship between problem-posing proficiency and dimensions of creativity, we found flexibility to be the strongest correlate of structured problem-posing proficiency. Flexibility is recognized as a critical cognitive component of creativity (e.g., Cai & Hwang, 2002; Kontorovich et al., 2011; Leikin & Sriraman, 2022; Silver, 1997; Yuan & Sriraman, 2011). This dimension necessitates exploring a problem from multiple perspectives and formulating diverse solution strategies (Torrance, 1962/1974). The strong presence of flexibility in structured problem-posing contexts indicates that PMTs are capable of developing alternative perspectives on real-world problems. Waynberg and Leikin (2012) emphasize that creativity-oriented activities in enriched learning environments foster students' fluency and flexibility. Therefore, increasing PMTs' awareness of mathematical structures, problem complexity, solvability, real-world relevance, and data quality may also contribute positively to their flexibility.

We identified originality as the most significant correlate of semi-structured problem-posing proficiency. Originality is defined by unconventional and infrequently encountered ideas (Torrance, 1962/1974). This phenomenon can be analyzed from two distinct perspectives. First, semi-structured problem-posing fosters the development of innovative perspectives and demands advanced conceptual knowledge, relational reasoning, and extensive information processing (Stoyanova & Ellerton, 1996). In this context, originality stands out as a unique and infrequent trait. Thus, given the inherent complexity of semi-structured problem-posing, students must cultivate advanced divergent thinking skills. This cognitive challenge compels them to generate more distinctive and unconventional problem content. Among the strongest correlations observed in problem-posing proficiency was that between flexibility and originality. The strong correlation between these two creativity dimensions suggests that problems generated without constraints tend to be more fluid and original. Free problem-posing facilitates the development of problems grounded in real-world contexts, free from external limitations (Baumanns, 2023). Indeed, in our study, we found that PMTs' problem-posing proficiency, along with their fluency, flexibility, and originality, was more prominently expressed through free problem-posing activities. This highlights the potential of free problem-posing as a powerful medium for nurturing creative mathematical thinking. Encouraging such open-ended tasks may therefore play a key role in enhancing pre-service teachers' ability to generate diverse and original mathematical problems. Therefore, integrating free problem-posing tasks into teacher education curricula may significantly contribute to the cultivation of creative potential in future mathematics educators.

Conclusion

Among the three types of problem-posing, PMTs demonstrated the highest level of problem-posing proficiency in free problem-posing. A restructured course curriculum contributed to a significant majority of the participants' achievement of high problem-posing proficiency levels. This outcome was largely influenced by their understanding of the theoretical foundations of RME and their awareness of the components of creativity. These factors had a particularly pronounced impact in the context of free problem-posing. In contrast, only half of the PMTs demonstrated high problem-posing proficiency in semi-structured problem-posing, while most of the remaining participants showed moderate proficiency. This finding indicates that semi-structured problem-posing proficiency has room for further development.

Additionally, some PMTs exhibited low problem-posing proficiency in structured, semi-structured, and free problem-posing. Our findings highlight the potential of free problem-posing tasks as powerful tools for nurturing mathematical creativity in teacher education. One of the key findings of the study is the relationship between originality in semi-structured and structured problem-posing processes. A considerable number of PMTs exhibited high problem-posing proficiency in semi-structured problem posing, while slightly fewer demonstrated high proficiency in structured problem posing. Although this finding was unexpected, it aligns with established theoretical perspectives on originality (e.g., Cai & Hwang, 2002; Silver, 1994; Yuan & Sriraman, 2011). Since originality inherently entails distinctiveness, this suggests PMTs tend to generate more original ideas in semi-structured problem-posing contexts.

Recommendations

Although problem posing has been considered a relatively neglected research area compared to problem-solving, educators and researchers have shown growing interest in this field in recent years (Cai et al., 2015). However, with advancing technology, the need for research on problem-posing has been steadily increasing. In this context, design-based studies incorporating technology can facilitate a deeper analysis of creativity indicators in problem-posing processes. This study promoted collaborative learning by initially emphasizing group work. However, problem-posing tasks were completed individually, producing individual outcomes. Future studies could incorporate group-based evaluations to examine the impact of similar design-based courses across different group settings. Another limitation of this study is the challenge of evaluating problem-posing processes multidimensionally. This study is based on the experiences of a group of Turkish students. Therefore, future research could investigate the impact of similar course designs in different cultural contexts. Additionally, leveraging technological advancements could facilitate enhancements across different dimensions of creativity (e.g., usefulness, sustainability, transferability, transformability), contributing to the development of a theoretical framework.

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