Studying Geometric Concepts in Elementary School through Construction by Compass and Straightedge

Khayriah Hasan Massarwe
The Arab Academic College for Education in Israel - Haifa, Israel

To cite this article:
Studying Geometric Concepts in Elementary School through Construction by Compass and Straightedge

Khayriah Hasan Massarwe

Abstract

Students regard geometry a difficult study subject, towards which they develop over time negative attitudes and low study motivation. Experiential and creative studying experience increases students' motivation and develops positive attitudes towards the study subject. Knowledge construction takes place through activity and making of tangible interest. This is a studying technique that supports the cognitive construction process of the student. Construction with a compass and straightedge in geometry can meet these requirements. Based on these principles a study was conducted among 50 elementary school students in the sixth grade from the Arab sector in Israel. The aim of the study was to examine the effect of combining geometric construction activities using a compass and straightedge in a geometry class on students' achievements, attitudes, and motivation. When the subject is not part of the school curriculum. The study also examined the skills needed to teach the subject and the difficulties involved. The findings of the present study showed that students who studied geometric concepts from construction with compass and straightedge raised achievements and motivation. They also developed positive attitudes towards geometry. Studying by compass and straightedge helped students to overcome difficulties at the geometric thinking levels, but it required dealing with new studying skills which piqued their curiosity for studying.

Keywords
Geometric constructions
Motivation
Attitudes

Introduction

Geometry is considered one of the major fields in mathematics and is taught as one of the core subjects in school. However, it is perceived by students as boring and mechanical for studying and the attitude towards it in most cases is negative (Shields, 2005). It is a study subject that poses various challenges to students such as difficulty in understanding geometric language and using it to formulate theorems, definitions, or relations between shapes (Gal & Linchevski, 2010). It further poses the difficulty involved in the skill of marking and sketching for the purpose of understanding the properties of geometric forms and the relations between them (Hoffer, 1981). In this setting, many students studying geometry achieve low results (Lehrer, Jenkins & Osana, 1998).

The combination of teaching methods of the study subject, the tasks required of students and their difficulty, accompanied by low achievements, impairs many of the students' confidence and self-esteem, and lowers the
motivation to study this subject, causing many of them to take negative attitudes towards geometry (Rozgonjuk, Kraav, Mikkor, Orav-Puurand & Űht, 2020). Studies in teaching geometry have highlighted the importance of finding ways to connect students to geometry and provide them with opportunities to experience exploration and discovery (Jirout, Vitiello & Zumbrunn, 2018).

According to the constructivist theory, studying is an active and continuous process through which the student constructs knowledge from the environment in which he studies and assigns personal meaning, relying on previous knowledge and experience (Dunham, et al., 2002). The constructionist theory proposes a teaching approach that supports studying through the practice of constructing a tangible product to increase their motivation (Papert & Harel, 1991). Hence the advantage of an activity that combines various studying tools, which involve tangible and visual action. One of the tools that can be used to create a tangible product is the use of a compass and straightedge in geometric constructions. The compass and straightedge make it possible to create a visual product, while preserving the properties of the shapes defined during construction (Sanders, 1999). Based on the mentioned above, a study was conducted among elementary school students in the sixth grade in the Arab sector in Israel (Taha – Heeb, 2021), when compass and straightedge is not part of the elementary school curriculum. The aim of the study was to examine the combination of geometric constructions using a straightedge and compass in a geometry class, and their impact on the achievements, attitudes, and motivation of sixth graders. The study also examined the skills needed to teach the subject and the difficulties involved.

**Theoretical Framework**

Geometry is a branch of mathematics that deals with shapes and structures, and defines terms such as lines, surfaces, curves, circles, and points (AL-salhat, 2022; Creager, 2022; Greenberg, 2008). It is a mathematically branch that is logically organized and is based on a formal system with agreed signs and symbols. It serves as a means of illustrating the physical world and assists in the development of various fields, such as art, architecture, chemistry, physics, and so on, which require adaptation and the use of design and prediction skills, alongside mathematical, creative and logical thinking (Janicic, 2009). Although geometry is a study subject that is important to teach in school, it is identified as a difficult study subject for both students and teachers. Students and teachers believe that this is a study subject in which the level of requirements is high (Leal, 2007). Formal geometry is considered a highly complex field that causes teaching and studying difficulties of the study subject. Many studies (Patkin & Dayan, 2012; Sarfaty & Patkin, 2013) point to difficulties with geometry among students; difficulties with identifying geometric shapes and recognizing features, difficulties that arise at an early stage and continue even in older students and even among teachers.

Many students encounter difficulties with regards to the four levels of geometric thinking according to Van Hiele's theory (1999). Hoffer (1981) argued that in order to understand the field of geometry, it is necessary to demonstrate a number of skills that students have difficulty mastering, for example visualizing abstract concepts. Among many students, their visualization ability is not sufficiently developed, and therefore they have difficulty studying the subject. Verbal expression is also important, as geometry requires the use of precise language to formulate theorems and definitions, and many students do not understand the formal language, lack reading
comprehension or wording, and therefore have difficulty with this study subject (Clements & Battista, 1992). In addition, geometry demands the ability of logical thinking, reasoning and distinction between cause and effect. Many students demonstrate deficiencies in their ability to distinguish between causes and presentations, and inability to identify justifications, making it difficult for them to solve problems in geometry (Sanders, 1999). Finally, it seems that in addition to the difficulty in acquiring concepts, the difficulty in creating the concept images, and the lack of basic skills, there is also a gap between the level of understanding of the student, and the level of teaching of the teacher. For example, it has been found that teachers usually focus on teaching the concepts in an abstract way, while the students in turn still try to understand the concept on the realistic level. This gap is another explanation for the reasons why students do not understand the material being taught. These difficulties, affect the quality of studying and impair students' self-confidence, and lower their academic motivation, causing many of them to take negative attitudes toward geometry (Rozgonjuk, et al., 2020).

**Motivation in Studying Geometry**

The term motivation refers to processes within the person that determine the direction of the behavior. High motivation is evident in investing a lot of effort over time to achieve defined goals (Ryan & Deci, 2000). In the field of education, motivation is an essential component in the studying process (Pintrich & Schunk, 2002). Students tend to adopt different goals for their academic endeavors. Some fulfill the task because they want to study, understand, develop their ability, or improve. Others fulfill the task because they want to show high ability and get high achievements. Namely, motivation can be internal that driven by control and self-perception of personal ability to study, or external in which it’s driven by the need to be accepted in society and receive feedback and support from the teacher (Bishara, 2018). Studying becomes significant for the student if it occurs out of intrinsic motivation, i.e., out of interest and curiosity. The purpose of the educational process is to provide an opportunity to arouse curiosity among students, from the perception that the student is a person with potential for curiosity, intellectual interest, and self-direction. The studying process becomes more meaningful for an active student, trying to achieve goals and produce meaning. Significant studying is such that constructs challenges, a sense of competence and motivation for new studying. Thus, the material studied, is perceived by the student as challenging and intriguing. This is how a process is created that calls for an experience of cognitive, emotional, and social growth.

Studies in mathematics education have pointed out geometry as one of the study subjects which teaching methods and goals do not encourage students' motivation to study, and therefore alternative teaching methods should be taken (Abramovich, Grinshpan & Milligan, 2018). Geometry begins with placing problems in the context of tangible activities. Similarly, researchers have referred to terms such as actuality, tangibility, and relevance, as synonyms for student motivation when it comes to geometry studies (Abramovich, Grinshpan & Milligan, 2018). According to (Dagnew, 2017), since curiosity is the source of motivation to study, one should turn to curiosity when studying geometry and motivate students according to what arouses their enthusiasm to study something that is not necessarily perceived as enjoyable but necessary. It is a type of conceptual motivation through which students' curiosity is used as a central tool for problem solving.
Students' Attitudes toward Geometry

Studies have pointed out geometry as one of the subjects perceived in a negative way. Hoffer (1981) argues that most students perceive studying as an automated process, in which they are primarily concerned with memorizing evidence in a formal, tedious, and incomprehensible manner. It has also been found that teachers have to more emphasis on developing important skills on which geometry is based. Accordingly, to make geometry studies be meaningful for students, geometric activities must be performed independently such as drawing and exploration. Further study has presented recommendations to make teaching geometry more meaningful and satisfying if it would include elements of exploration and discovery (Leikin & Lev, 2012). Students should be allowed to raise questions and hypotheses through discovery and exploration, as this can encourage them to create new and original ideas. The combination of all the skills and recommendations listed above, in the teaching of geometry, will contribute to increasing the student's interest with this study subject, and his understanding of the material studied. The connection between academic achievement and students' attitudes is a reciprocal connection. When the student exhibits positive tendencies to studying, he succeeds in reaching various dimensions of intellectual depth, self-management ability and self-knowledge, thus reaching high academic achievements (Pavlovicova & Zahorska, 2015). Studies indicate a relation between students' attitudes and their motivation for studying and the impact of this relation on their level of achievement. In a study that examined student success, it was found that there are significant relations between students' attitudes toward school, their motivation, and academic achievement (Jirout & Klahr, 2012). One of the most important factors that leads a student to reach his goal is motivation. This motive is motivation for achievement, which shapes and determines the attitude direction and the behavior expressed in its wake (Deci & Ryan, 2012).

Teaching Geometry

Studies in mathematics education suggest that teaching geometry by connecting it with the student's real life naturally evokes his motivation (Abramovich, Grinshpan & Milligan, 2018). To understand new knowledge, students need to combine understanding and experience and imagination and reasoning. They must use existing mathematical knowledge to solve problems unfamiliar to them (Jirout & Klahr, 2012). Solving problems in several ways is a tool for developing mathematical creativity. Creative teaching emphasizes the involvement of students in the constructive mathematical process, encourages autonomous studying and considers differences between students. The teacher has a central role in the development of the mathematical creativity of his students, and he should encourage and direct them to self-studying. The development of cognitive and meta-cognitive skills is a necessary condition for meaningful studying which leads to creative thinking (Renesse & Ecke, 2017).

According to constructivist theory, studying is an active and continuous process through which the student receives information from the environment in which he is studying and assigns personal meaning, relying on previous knowledge and experience (Dunham et al., 2002). This approach emphasizes the importance of the active student constructing the knowledge by acquiring new concepts, and at the same time incorrect concepts take on new meanings. Hence studying is a dynamic process of acquiring new concepts and eliminating concepts created in the student intuitively and cognition undergoes changes of reconstruction. And according to the constructionist...
approach, the construction of knowledge takes place during a relevant construction activity with a tangible interest, studying from activity and doing. It is a type of studying that supports the cognitive construction process of the student (Papert & Harel, 1991). Under this conception, there has been a change in teaching methods that have included changes in the components of the traditional studying environment in which the teaching of geometry takes place. Technological tools have been found to be unique tools that help increase students’ motivation, as it is a tool, they make extensive use of in their lives (Schettino, 2016). A technology-intensive environment reinforces the value of studying tasks, the meaning of the content being studied and the development of geometric thinking. Hence the integration of technological tools in the teaching of geometry is important and even guarantees students an upgraded studying experience. The education system needs to adapt to the technological changes that have affected society and culture and train students for future functioning in a digital world, which has led to changes in the perception of teaching methods - studying in school, defining new skills to develop in the student, and finding teaching tools with which to achieve them (Eldar & Shrieber. 2016). A technology-intensive environment reinforces the meaning of the content studied and the development of geometric thinking. Hence, the integration of technological tools in the teaching of geometry is important and even guarantees students an upgraded studying experience (Janicic, 2009). Children are inherently curious and are actively involved in constructing and developing their knowledge. Hence the advantage of an activity that combines tools such as a compass and straightedge, in which the student constructs his knowledge from geometric construction.

**Compass and Straightedge in Geometry Class**

The compass is a tool for drawing circles and arcs. The geometric straightedge is a tool, which allows the creation of a straight line or a long section, passing through two given points. The straightedge is not calibrated and has no measurement abilities because it does not have marked scale marks, which indicate units of length, and it is assumed that it is not possible to create with the straightedge alone two parallel lines at a given distance from each other (Sanders, 1999). In geometry the non-measuring straightedge and compass are tools that can be used to solve construction problems that meet the premises of geometry, as reflected in its foundations. In Euclidean geometry of the plane, construction with a compass and straightedge is the construction of geometric objects, such as segments with definite properties, that use only these tools. In this respect, the straightedge and compass are not the physical tools used in the drawing, but geometric simplifications.

In Sanders’ (1999) study, to encourage visualization among students, they used a compass and a straightedge to create geometric structures to make studying more tangible. Students then experience discovering the properties of the geometric shapes created from construction and proving them. It turned out that there was an increase in motivation in studying geometry among students who used tools like compass and straightedges and enjoyed studying geometry and sketching schemes and their designs.

The present study examined how geometric constructions using a compass and a straightedge, affect the achievements, attitudes, and motivation of a sixth grader. Students studied topics of construction using a straightedge and compass such as: constructing an isosceles triangle, crossing a section, intersecting an angle, etc.
In this way the students gained their knowledge from constructing a product like intersecting an angle for example he can measure and check that indeed the measurements are correct and that the activity he performed in intersecting an angle does indeed lead to a correct result. The use of a compass and straightedge contributes to the encouragement of visuality among students as it includes visual representations such as images, presentations, encouraging the sense of sight (Sanders, 1999). It can make studying more tangible, which may contribute to an increase in students' motivation to study, a positive change in their attitudes about the study subject and an improvement in achievement.

**Research Questions**

1. What is the effect of the use of compass and straightedge in geometric constructions in a geometry class on motivation, attitudes, and achievements among sixth graders?
2. What are the difficulties with teaching geometry using a compass and straightedge?
3. What are the basic skills needed to teach geometry using a compass and straightedge?

**Methodology**

The present study is a quantitative and qualitative combined study. The quantitative part is designed to examine the impact of studying by compass and straightedge on the attitudes, motivations, and achievements of the students. It included quantitative data to answer the first two study questions, and statistical analysis of students' attitudes, motivation, and achievements. A qualitative interview and study log (Taha – Heeb, 2021) are intended to deepen the results in an interpretive verbal way, thus reinforcing the validity of the quantitative data obtained. For the purposes of this work, a correlation study layout was used to examine the relation between variables. This layout examined how the dependent variable (use of a compass and straightedge) affects the independent variable (motivation, achievement, and student attitudes). In addition, a qualitative layout was used, that helped to examine the context interpretively to deepen the meaning of the connection between the use of the compass and straightedge, and the motivation, achievements, and attitudes of the students.

**Participants**

50 sixth graders from an elementary school in the Arab sector in Israel participated in this study. The school has two heterogeneous sixth graders. Both classes study the same geometry study unit with the same teacher (Taha – Heeb, 2021). The students were divided into two groups: Experimental group and control group. To create two similar groups in terms of school achievement, a preliminary knowledge exam was conducted for all sixth-grade students in order to assign them to groups.

**Instrumentation**

In the present study, five main tools were used: Two qualitative ones, study diary and interview, and three quantitative ones. The researcher diary included a document of the actions the teacher performed during the
lessons throughout the study, the experience, difficulties, and insights for further research. These impressions helped in understanding the difficulties involved in teaching the study subject, as well as teaching the lesson using a compass and straightedge. A semi-structured interview was also used and was conducted with ten students, and included questions related to the various difficulties that arose while studying geometric construction using a compass and straightedge, as well as the pros and cons versus frontal studying. For example, the students were asked: What were the difficulties they faced while studying the subject and how did they deal with them? What do they think are the pros and cons of the subject? In their opinion, what are the differences between frontal instruction and instruction with the help of a compass and a straightedge?

A preliminary exam was conducted for both classes in the material being studied to determine the student achievements. The motivation and attitude questionnaires were developed and examined to assess their suitability for sixth grades by math teachers at the same school the research conducted. Students answered the questionnaires before and after the study unit. They included closed-ended verbal questions with choices from the Likert scale. Since the questionnaire is presented to sixth grades, the scale grading was set to a scale between 1 = do not agree at all and 3 = strongly agree, so that the students will not be confused. The motivation questionnaire dealt with students’ preferences regarding studying and the behaviors related to it. The questionnaire included 15 relevant statements, the items dealt with various areas related to studying and represent high or low motivational patterns in terms of quality and examine to what extent does the student expresses a willingness to demonstrate each of them. The reliability of the questionnaire according to Alpha Kronbach in the first presentation: $\alpha = 0.723$. The reliability of the questionnaire according to Alpha Kronbach in the second presentation: $\alpha = 0.87$.

The positions questionnaire items address number of topics in the cognitive field, and in the effective field, with the student being asked to give an opinion on the geometry study subject and its importance, the teaching method, the achievements, and the attitude to the subject. The score for each claim was given so that a maximum positive claim received 3 points, and a minimum positive claim (negative position) received only one point. The reliability of the questionnaire according to Alpha Kronbach in the first transfer: $\alpha = 0.875$. The reliability of the questionnaire according to Alpha Kronbach in the second transfer: $\alpha = 0.828$.

**The Study Process**

In the first stage, all the students were tested in knowledge exam in geometry. In the second stage, all the students answered the motivation questionnaire and the attitude questionnaire towards the geometry study subject. In the third stage, the students were divided into two groups - an experimental group and a control group. The control group studied in a frontal teaching format, and the experimental group studied, for the purpose of the study, the subject through integrating constructions using a compass and straightedge. The study unit on construction using a compass and straightedge included topics such as constructing an equilateral triangle, constructing an isosceles triangle, intersecting an angle, intersecting a section, etc. During construction an analysis was made of the geometrical shapes obtained at each stage and identification of their properties. Measurements were also made by the students to understand and identify the correctness of the construction with compass and straightedge. For example, with respect to intersecting an angle, the students measured the sketched with a protractor, as well as the
angles formed after intersecting it. With respect to intersecting a section with a compass and straightedge, they measured its length after intersecting the section with a ruler and verified that it was indeed cut into two equal sections. In the fourth stage, after the completion of the study unit, the two groups took the same re-examination. Motivation and attitudes questionnaires were also resubmitted after the exam. In the fifth stage, after answering the questionnaires, a semi-structured interview was conducted with students from the experimental group, about the use of a compass and straightedge, in order to examine the unique difficulties of the subject, its advantages and its disadvantages. Throughout the teaching of the study unit the teacher documented, the difficulties she encountered while teaching the unit, and what are the skills required for teaching and studying in this way.

**Data Analysis**

The data analysis in this study was done in two ways. First, the data collected from the quantitative tools from the exam and the two questionnaires were analyzed. In the quantitative study, the data collection process involved entering data into a table and encoding the data using various statistical software, which examine relations between variables. In the present study, students' grades in the exam and their questionnaire answers were all collected and coded in a table. Then, to examine the differences in the grades, attitudes and motivation of the students, an exam was held to examine the relation between the variables t-test for dependent samples. This test was used twice - the first time it examined the differences in the grades, attitudes, and motivation of the students, before the study unit, and the second time it examined after the study unit.

To deepen the information about the statistical analysis obtained on the attitudes, motivation and achievements of students, a qualitative analysis was also performed in the present study. The purpose of the qualitative data analysis process is to give deep meaning or further interpretation to the phenomenon. In the present study, an interpretive analysis was made of the teacher's notebook, and of the interviews with the students. The analysis was based on a categorical division method, according to which key topics that were repeated or found in common among the study participants were removed from the interviews and in the diary, and these were divided into main categories and main themes. The analysis was done in seven steps. In the first stage, the interviews and the diary were read repeatedly until they were internalized and recorded alongside initial ideas. In the second stage, the interviews were divided into meaningful units, that is, sections of the interviews. In the third stage began the coding phase of the meaning units, according to which each unit was given a title. In the fourth stage, all the titles that were found to be related to each other were categorized into categories and sub-categories. In the fifth stage, an open coding of the categories was performed, in which the categories were divided into themes. In the sixth stage, a transition was made to “axial coding”, in which all the categories from all the interviews and quotations were gathered into one file to distinguish between the differences and the similarities between them. In the seventh and final stage, the relations between the various themes were theoretically conceptualized.

**Findings**

The results of the study include two parts. The first part presents the descriptive statistics of the study variables while the second part presents the examination of the study hypotheses.
Descriptive Statistics

Table 1 lists the center indices and distribution indices of the study variables. Table 1 shows that for both groups, exam scores were higher after studying the study unit than before it. This is also true for the experimental group where an increase was found in students’ attitudes toward geometry and their motivation to study geometry after the study unit. In contrast, for the control group in the second presentation of the questionnaires, there is a decrease in their motivation to study geometry.

Table 1. Center and Distribution Indices of the Study Variables (N = 50)

<table>
<thead>
<tr>
<th>Stage</th>
<th>Group</th>
<th>Variable</th>
<th>Average</th>
<th>Standard deviation (sd)</th>
<th>Median</th>
<th>Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before a study unit</td>
<td>All students</td>
<td>Exam score</td>
<td>70.32</td>
<td>12.13</td>
<td>70</td>
<td>55</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Attitudes towards geometry</td>
<td>1.86</td>
<td>0.50</td>
<td>1.71</td>
<td>1.29</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Level of motivation</td>
<td>2.12</td>
<td>0.25</td>
<td>2.15</td>
<td>2.15</td>
</tr>
<tr>
<td>After study unit</td>
<td>Control</td>
<td>Exam score</td>
<td>72</td>
<td>11.44</td>
<td>70</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Attitudes towards geometry</td>
<td>1.95</td>
<td>0.38</td>
<td>1.85</td>
<td>1.86</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Level of motivation</td>
<td>1.65</td>
<td>0.16</td>
<td>1.69</td>
<td>1.77</td>
</tr>
<tr>
<td></td>
<td>Experimental</td>
<td>Exam score</td>
<td>79.28</td>
<td>11.74</td>
<td>78</td>
<td>75</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Attitudes towards geometry</td>
<td>2.64</td>
<td>0.27</td>
<td>2.57</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Level of motivation</td>
<td>2.33</td>
<td>0.39</td>
<td>2.46</td>
<td>2.69</td>
</tr>
</tbody>
</table>

The first study hypothesis was that teaching with the use of a compass and straightedge, helps identify geometric shapes and thus promotes student achievement in geometry more than with traditional teaching. The results of the study indicated differences in the scores of the examinees in geometry, between the average of the first exam taken before the study unit, and the average of the second exam taken after the study unit. To inspect whether there are differences in the mean point averages before and after the presentation of the study unit, a t-test was performed for dependent samples. For the control group the test showed that there were no significant differences in the mean score (t (24) = -0.682, p > 0.05), between the average of the first exam (mean = 69.8, sd = 11.8), and the mean of the second exam (mean = 72, sd = 11.44). For the experimental group, the test showed that significant differences were found between the average scores of the two exams, (t (24) = -2.395, p <0.05), where the second exam average (mean = 79.28, sd = 11.74), was significantly higher than the average of the first exam (mean = 70.84, sd = 12.7).

To inspect whether there are differences in the mean scores between the control group and the experimental group after studying the study unit, a t-test was performed for independent samples. The test showed that significant differences were found in the mean scores (t (48) = -2.22, p <0.05), between the exam mean of the experimental group after a second presentation (mean = 79.28, sd = 11.74), and the exam mean of the control group after a second presentation (average = 72, sd = 11.44). The findings indicate that the first study hypothesis was confirmed.
The second study hypothesis was that the use of a compass and straightedge creates positive attitudes toward geometry. The results of the study indicated differences in the attitudes of students towards the study of geometry before the learning of the study unit and after the learning of the unit. To inspect whether there are differences in students' attitudes toward geometry before and after the study of the unit, a t-test was performed for dependent samples. For the control group, the test showed that there were no significant differences in the students' attitudes towards geometry (t (24) = -0.149, p > 0.05), before the study of the unit (mean = 1.93, sd = 0.43), and after the study of the unit (mean = 1.95, sd = 0.38). For the experimental group, the test showed that significant differences were found in the students' attitudes towards geometry (t (24) = -7.14, p < 0.01), where the students' attitudes after the study unit (mean = 2.64, sd = 0.27) were significantly higher than the students' attitudes before the presentation of the study unit (mean = 1.78, sd = 0.55).

To inspect whether there are significant differences in students' attitudes toward geometry between the control group and the experimental group after the presentation of the study unit, a t-test was performed for independent samples. The test showed that significant differences were found in the mean scores (t (48) = -7.33, p < 0.01), between attitudes towards geometry among the experimental group after a second presentation (mean = 2.64, sd = 0.27), and attitudes towards geometry among the control group after the second presentation (mean = 1.95, sd = 0.38). The findings indicate that the second study hypothesis was confirmed.

The third study hypothesis was that the use of a compass and straightedge engages in the construction of a tangible product which increases students' involvement and motivation in studying. The results of the study indicated differences in the motivation of students to study geometry between before and after studying the study unit. To inspect whether there are differences in the motivation of students to study geometry before and after the presentation of the study unit a t-test was performed for dependent samples. For the control group, the test showed that significant differences were found in the motivation of the students to study geometry (t (24) = 5.58, p < 0.01), before studying the unit (mean = 2.09, sd = 0.31), and after it (mean = 1.65, sd = 0.16), and in fact there is a decrease in the level of motivation. For the experimental group, the test showed that significant differences were found in the motivation of the students to study geometry (t (24) = -2.08, p < 0.05), where the motivation of the students after the study unit (mean = 2.33, sd = 0.39), was significantly higher than the motivation of the students before that (mean = 2.15, sd = 0.19).

To inspect whether there are significant differences between the control group and the experimental group in the motivation of the students to study geometry after the study of the unit, a t-test was performed for independent samples. The test showed that there were significant differences in the motivation to study geometry (t (48) = -8.01, p < 0.01), between the motivation of the experimental group after the second presentation (mean = 2.33, sd = 0.39), and the motivation of the control group after the second presentation (mean = 1.65, sd = 0.17). The findings indicate that the third study hypothesis was confirmed.

Findings from the Interviews

This chapter presents the main findings that emerged from the analysis of the interviews with students and their
comparison with the data from the teacher diary. The analysis of the interviews yielded four main themes. The first theme: Difficulties with studying geometry - describes the various areas in which students find it difficult to study geometry. The second theme: The contribution of the use of compass and straightedge in studying geometry. The contribution of using compass and straightedge in geometry class to overcome difficulties. The third theme: Difficulties associated with using a compass and straightedge - describes the unique difficulties associated with using a compass and straightedge. Finally, the fourth theme: Change in students' attitudes towards the study subject of geometry following studying using a compass and a straightedge - describes whether and how some of the students' attitudes towards the study of geometry have changed, following the use of a compass and straightedge. In Table 2, the summary of the four themes is provided.

Table 2. Themes emerged from the Interviews

<table>
<thead>
<tr>
<th>Theme</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Difficulties with studying geometry.</td>
<td>Difficulties with visual skills.</td>
</tr>
<tr>
<td></td>
<td>Difficulty with mathematical literacy.</td>
</tr>
<tr>
<td></td>
<td>Difficulties with using correct mathematical language.</td>
</tr>
<tr>
<td></td>
<td>Difficulties with understanding geometric definitions and theorems.</td>
</tr>
<tr>
<td></td>
<td>Difficulties with the applied logical aspect.</td>
</tr>
<tr>
<td>2 The contribution of the use of compass and straightedge in studying geometry.</td>
<td>Assistance in understanding the meaning of abstract geometric terms and concepts.</td>
</tr>
<tr>
<td></td>
<td>Development of self-studying ability.</td>
</tr>
<tr>
<td></td>
<td>Improving verbal language skills.</td>
</tr>
<tr>
<td></td>
<td>Development of the ability to solve geometric problems.</td>
</tr>
<tr>
<td></td>
<td>Assistance in developing marking and construction skills in geometry.</td>
</tr>
<tr>
<td></td>
<td>Understanding properties of shapes, and relations between shapes</td>
</tr>
<tr>
<td></td>
<td>Improving the ability to reason</td>
</tr>
<tr>
<td>3 Difficulties discovered among students when using a compass and straightedge.</td>
<td>Technical difficulties with the absence of experience - incorrect use of the compasses</td>
</tr>
<tr>
<td></td>
<td>Logical-applicable difficulties.</td>
</tr>
<tr>
<td></td>
<td>Difficulties arising from a long process of construction and multi-stage.</td>
</tr>
<tr>
<td>4 Change in attitudes towards the study subject of geometry following studying using a compass and a straightedge.</td>
<td>Students now see geometry as an intriguing study subject compared to what they used to see as a boring study subject.</td>
</tr>
<tr>
<td></td>
<td>Students have become more active.</td>
</tr>
<tr>
<td></td>
<td>Raising motivation.</td>
</tr>
</tbody>
</table>
**Difficulties with Studying Geometry**

The interviews with students indicated that they encountered various difficulties about studying the geometry study subject. For example, from the students' descriptions, it seems that some of them encountered difficulties with basic skills:

- **Difficulties with visual skills.** Students had difficulty seeing the various concepts and theorems, which were mostly abstract:

  > “It was harder for me to understand in the instructions what I should do with this form. I could not read how the instructions say it should be done and write according to ... really how to do it in the notebook.

- **Difficulties with using mathematical language.** Difficulty verbally expressing definitions, shapes, formulations, and arguments:

  > “I completely failed to understand the exercises and what should be done. I could read but did not understand what to do. Say if it was written to draw a triangle ... I would be really confused by the definitions. An isosceles or equilateral? What is the difference between height, median or perpendicular? It’s sounded the same. Then many times I would not understand the question at all. The skilled researcher expressed that student lack a basic understanding of concepts. Failed to detect differences in features of equilateral and isosceles triangle. Or the sum of the angles.

- **Difficulties with mathematical literacy.** Difficulty understanding requirements:

  > “It's hard for me to understand the instructions sometimes or understand what it is. So, I need help from the teacher who will explain to me in a different way. well, know what each thing means and then draw it as they asked so I did not always understand what they asked for. Only after the teacher explained, I did understand.

- **Difficulties with the applied logical aspect.** It seems that most of the students are still at the first level of thinking of Van Hila which is the level of identification. Because they have noted how they identify the shapes in general, their structure and know how to distinguish differences between them, but they still do not know how to separate the components of the shape and treat them separately. They defined the shapes according to the general shape and do not know how to identify the properties of that geometric shape:

  > “It was easy to know the differences between the shapes, but the hardest part for me was to explain why the shape is called that. How I came up with a solution. Let's say many times I just knew how to say it is a parallelogram, but I did not know to say why it is a parallelogram and not rhombus.

- **Difficulties in understanding geometric definitions and theorems.** Many students have mentioned how, many times they remember wordings of theorems without understanding the intent behind them:

  > “It's harder in my opinion to simply understand what's being taught in class. It feels like the lesson is dry. The exercises are boring. Sometimes I just know what the answer is as I already memorize the definition but not because I understand what it says. I just know that this is that. Because I studied it by heart.
The Contribution of the Use of Compasses and Straightedges in Studying Geometry

Throughout the interviews, students described how the use of a compass and straightedge helped them deal with various difficulties encountered in studying geometry. From the researcher's diary it was possible to see whether the students' descriptions and perceptions about the use of the compass and straightedge were related or contrary to what was written by the teacher when he was teaching in this way.

- **Understanding the meaning of abstract terms and concepts.** It allowed them to visually see the shape more concretely:
  
  *For me it helps more to understand the concepts. Explore them while sketching. It helps me remember features because I construct them. I already know how to recognize them and then it is also easier for me to understand what is being done in the exercise because I already recognize everything that is written in the instructions ... It helps me more to imagine the shape in my head.*

- **Developing the capacity for self-studying.** The use of a compass and straightedge helped them understand how to study independently, but still with teacher guidance.

- **Improving verbal language skills.** The use of compass and straightedge helped students expand their vocabulary in geometry. For example, some students describe how the activity allowed them to understand the various concepts they had difficulty understanding in class:
  
  *With the compass and straightedge, you do not have to remember or study by heart. You can construct and then just see it. "Suddenly it reminds me of other things I already studied or did, but only after drawing did, I suddenly also understand what I studied.*

- **Understanding geometric concepts.** Students pointed that studying with compass and straightedge helped them to suddenly understand materials they had already studied before, when they had to memorize concepts orally without trying to understand the logic:
  
  *... it's more helpful to suddenly understand a sentence I copied from the board and did not really understand it, but then we studied with compass and straightedge, suddenly it appeared to me” ahhh ...now I understand what the teacher meant. " Once I had to draw a perpendicular, or construct a shape myself, I suddenly remembered the material better because after I sketched it, it all of a sudden .... I understood the meaning of the definition, and I remembered, let’s say, the differences, between the features, so it was also easier for me after that to understand the questions in class.*

- **Assistance in developing marking and construction skills.**
  
  *We not only copy we also draw on our own. .. Because with the compass and the straightedge you are not told construct a parallelogram. You are actually given step-by-step instructions on how to draw and then suddenly you see that you have a parallelogram.*

- **Understanding the properties of shapes, and the relations between shapes.**
  
  *I could also see with the instructions, what its features are. And it was easier to understand and see.*
- Development of the ability to solve geometric problems. According to the students, the activity helped them to understand more clearly the instructions and requirements of the geometric problems. An opposite picture emerged from the researcher's diary, according to which he finds that the students had difficulty understanding the instructions of the exercises, as they had difficulty following what was required at each stage of the exercise. They did not understand the instructions for the construction but under the guidance of the teacher.

- Improving the ability to reason. Defining and identifying features of geometric forms, which allowed the students to justify the differences between them:

  The straightedge and compass helped me explain the differences between the shapes. Why a rectangle and square are different. Not just to identify and say here is a rectangle and here is a square. But to know what the differences are between them.

Difficulties associated with the Use of a Compass and Straightedge

Along with the many benefits of using a compass and straightedge in geometry class, students have encountered various difficulties.

- Technical difficulties. Related to how to use the compass because this is their first experience. It was difficult at first to hold it correctly and use it to draw accurately as required:

  At first it was a little hard to hold the compass and figure out how to draw with it. And in the beginning, it came out crooked. "It took me a while to study how to hold it ...". "... just figure out how to do with the pencil and compass, draw. The circle came out wrong. And you have to delete many times and redraw it to be accurate."

This is consistent with the diary of a researcher-teacher who indicated difficulties among students in adapting to the technical work with the compass in the first lesson (see Figure 1).

![Figure 1. Technical Difficulties: The First-time Students used a Compass](image)

The right figure in Figure 1 depicts the first-time students used a compass. The figure to the left shows an example of their difficulty in drawing arches and connecting them to each other.
- **Logical-applicable difficulties.** Students had difficulty applying concepts they studied in the lesson in which they learned how to use the compass and straightedge, to other geometry lessons in which the material was studied without these tools:

  *I still sometimes fail to understand the homework the teacher gives. Like how to use a compass and a straightedge in homework. With a straightedge you can draw only lines and with a compass is just circles or arcs. There are things that the teacher gives where sometimes the instructions are different from the instructions in the exercises with the compass.*

Figure 2 shows an example of the difficulty of students following the instructions of the exercise and drawing the exact drawing. In the right figure you can see that due to the lack of skill in holding the compass, the students had difficulty creating the drawing accurately, which required them to make many erasures and create again. In the figure to the left, sometimes the students had to manually complete the drawing in order to be as accurate as required.

![Figure 2. Lack of Skill in Holding the Compass, Difficulty creating the Drawing Accurately](image)

- **Difficulties resulting from a long process of construction and multiple stages.** Students noted that studying with a compass and straightedge involves many steps, which need to be followed in order, which is perceived by many as a long and sometimes tiring process:

  *At first I had a hard time keeping up because there are a lot of actions. And if it was written to draw two arcs from the same point, I did not understand what that meant. Then I would come to the next stage, but it could not be done because it dependent on the previous stage that I did not do well. And I had to go back.*

- **Difficulties with understanding mathematical language.** Students expressed difficulties with understanding the language, and had difficulty understanding the instructions of the exercise and what they were needed to sketch:

  *... Sometimes I also did not really understand yet how to draw according to the instructions. There are a lot of exercises where I have a lot of sections that need to be done one after the other and sometimes, I do not do well and need to go back to correct. It is impossible to skip the order and it can be a little tiring at times. But it's better than standard studying...*
Figure 3 shows an example of the students' difficulty in understanding the instructions of the exercise and applying it in steps. The right figure illustrates the beginning of the student's asked to draw a perpendicular from point A that intersects the axis at point D, and then create an arc. The left figure depicts his final product, in which he appears to have misunderstood the instructions of the exercise, and that the points of intersection of the arches do not correctly express what is required.

![Figure 3](image)

**Figure 3. Difficulty Understanding the Instructions. Draw a Perpendicular to create an Arc**

*Changing Students’ Attitudes following Studying using a Compass and Straightedge*

The following is a description of comparing the positions before and after studying using a compasses and straightedge:

- *Geometry is a boring.* Before their experience with compass and straightedge students described the study of geometry as a boring and that their involvement in the class was passive without engaging thought processes:

  ... *in class you mostly sit and listen to the teacher. Not really doing much. Copy from the board and summarize what the teacher says, then study a topic and have homework...*

- *Active involvement in the class.* Studying geometry using a compass and a straightedge, made them more active partners in the studying process, and allowed them to activate thought processes, which improved their motivation to attend classes:

  .... *"It's nicer to do it in class with a compass and a straightedge, because it's not just the teacher standing and explaining on the board." "With the compass we create. We do not just copy ... "*, we construct a shape and then it is easier to identify because we do not just copy it. It's more fun and interesting to study like that.

- *Creating a visual product.* From their words they seem to enjoy studying using a compass and straightedge because this activity allows them to create something by themselves visually, which arouses in them the desire and curiosity to study:

  With a compass and straightedge, it's just more interesting. A lot of times then you have to mark in colors and write and construct the shape. And go in order. First draw a line. After those two points. After that draw a circle. I had more fun drawing the shape myself and not just getting it on a page and answering an exercise.
Figure 4 shows the improvement in the students' performances with the compass. The right figure shows control of motor skills in holding the compass precisely to draw a circle. The left figure depicts the student's final construction product, after acquiring motor skills and illustrates how he subsequently acquired the drawing skill as well.

Figure 4. Improvement in the Students’ Performances with the Compass and Acquiring Motor Skills

- *Increase in the level of interest and motivation in geometry.* Students described that their curiosity, motivation, and desire to attend classes had increased since they started studying with compass and straightedge. They pointed that this way of teaching is more interesting than the traditional form of studying, during which they study geometry:

  "... it's like it was indeed more interesting to study like that and not continue to study like we usually do. Studying with the straightedge and compass, so I sometimes waited for the lesson and wanted to participate. "... it was more interesting. More intriguing. I wanted to listen and do exercises. It was not as boring as I thought it would be."

In conclusion, the use of compass and straightedge, helped students deal with difficulties they faced in class, which led to a change in their attitude towards geometry and greater motivation for studying it. Along with the many benefits that the students noted in the context of studying with a compass and straightedge, there were also difficulties that arose during geometric construction. But unlike the difficulties that the students described as impairing their attitudes and motivation, the difficulties associated with the compass and straightedge were described by the students not as a difficulty but as a challenge, which aroused in them the motivation to overcome it and deal with the difficulty.

**Discussion**

This study examined the integration of geometric constructions using a compass and straightedge in a geometry class and the impact on the achievements, attitudes, and motivation of sixth-graders. The results of the study indicated an increase in achievements in the geometry study subject among students who experienced construction with the help of compass and straightedge compared to students from the control group. In the experimental group,
higher achievements were measured after studying the unit than before, and it was also found that the average grades of these students after studying the unit were even higher than the average grades of students studying with the traditional frontal teaching method. Similar results emerged from the interviews with the students. For example, the students described that the use of a compass and straightedge, helped them to imagine the geometric shape and understand its properties, which helped them to identify it more easily. They also described that the geometric construction using a compass and straightedge helped them to understand geometric concepts. From the aforesaid it is possible to deduce what is the geometry level typical to students in the present study, in accordance with Van Hiele's thinking levels theory (Van Hiele (1999). In the context of this finding, it seems that the compass and straightedge helped the students deal with difficulties at the first level of thinking, which is the visual level. Most students seemed to have difficulty understanding the visual detection stage and had difficulty demonstrating visual skills of abstract concepts. With regarding to the second study question, the results of the study showed that there was a change in the attitudes of the students from the experimental group before and after studying the unit, and they exhibited more positive attitudes towards geometry, than students from the control group, who still exhibited similar attitudes towards the study subject.

From the interviews, it can be said that students who have experienced geometric constructions with compass and straightedge, have discovered that the geometry study subject can be interesting and intriguing. Thus, it is an indication that students' attitudes have become more positive towards the study of geometry. This finding indicates that the studying method was more intriguing for them, and included their own study and self-work, which did not necessarily make use of unambiguous terms and formulations. This is in line with the findings of (Leikin & lev, 2012), who argued that in order to make the studying of geometry satisfactory and meaningful for students, teacher must act for the sake of exploration and discovery and allow students to explore constructions creatively. The use of a compass and straightedge, provides a study environment for students that allows them to speculate about the problem presented, and helps them see geometry in a new light, thus encouraging them to create new and original ideas.

The use of a compass and straightedge deals with the construction of a tangible product, which increases students' involvement and motivation to study. The results of the study show that in the group in which the students experimented with a compass and a straightedge, there was an increase in their level of motivation to study. These results were also reflected in the students' attitudes towards the geometry study subject. Students described the studying method with the help of a compass and a straightedge as active studying, which allows them to create, construct and illustrate constructions, shapes, and drawings, and in fact view the product they have created. Such studying allows them to examine and explore things independently (Dagnew, 2017). Throughout the interviews, students described how they enjoyed creating geometric shapes themselves using the compass and straightedge and found that they were more motivated to attend the class. This finding is consistent with Sanders (1999) who stated that the use of a compass and a straightedge encourages visuality among students, because it allows them to create geometric shapes in a more tangible way. In the present study there was an increase in motivation in studying geometry among the students who used these tools. An interesting and unexpected result that was found dealt specifically with the results of the control group that did not experiment with geometric constructions. It was found that although there was no change in the teaching method by which the group studied, there was a decrease
in its motivation. This also increases the gap between the positive motivation level of the experimental group, compared to the low motivation level of the control group. Regarding the question of the effect of the use of compass and straightedge in geometric constructions on the motivation and achievements of students in the geometry lesson, the findings indicated a relation between students' motivation level and teaching method (Abramovich, Grinshpan & Milligan, 2018), where studying with compass and straightedge increases the level of motivation of students. This is consistent with the principle of meaningful studying and the constructive approach, which argues for the importance of activity in studying, in that the student constructs his knowledge and gives new meanings to familiar terms.

The gap in the level of motivation of the control group may be due to the teaching method in which it studied, compared to the teaching method with which the experimental group studied. Hence it seems that similarly to the findings of (Renesse & Ecke, 2017), teaching geometry using a compass and straightedge has been found to be an effective way to increase students' motivation compared to the traditional method, as it enables a process of knowledge construction, through the activity of construction relevant to students of a tangible, personal and significant interest (Jirout & Klahr, 2012). It is an active studying, which supports the cognitive construction process among students (Papert, 1993).

The results of the question about the difficulties of teaching geometry using a compass and straightedge shows that apparently some of the students still had difficulty implementing the instructions, but most of them have mostly technical difficulties, relating to holding the compass. In the beginning they had difficulties in drawing the shapes but over time these difficulties diminished. In this context it can even find an answer to the question about the basic skills needed for teaching geometry using a compass and straightedge. Some of the skills that students need to study seem to be related to reading comprehension, but most of the skills needed to use a compass are motor skills - such as adapting to holding it and accurate drawing through using it.

The results of the study and its findings, receive a lot of study support. From the results of the current study it could be seen that most of the students exhibit difficulties with this study subject of geometry that was discussed in various studies, such as the difficulty in acquiring concepts, creating concept images, and lack of basic skills (Hoffer, 1981; Clements & Battista, 1992). Support has also been found for the study findings dealing with the use and acquisition of the language of geometry, and how it differs in its uses from everyday language (Van Hiele, 1999; Patkin & Dayan, 2012). It was found that most of the students in the present study, preferred to engage in activities in geometry that include the use of a compass and straightedge, because they have the possibility to perform geometry activities, such as drawing and inquire. This finding is consistent with the remarks of (Sarfaty & Patkin, 2013), who noted that in order to create a meaning of a concept for the students, they must be made active partners. It also emerged that students have difficulty understanding the abstract language used in geometry and studying by using a compass and straightedge helps students to simplify the language and adopt a simpler, less formal geometric and more of a day-to-day language. The compass and straightedge were found to be tools with which the problems of geometry can be studied flexibly, and different situations can be described with the help of models of a less fixed and formal language.
Conclusions

The present study showed that construction with the help of a compass and straightedge required students to exhibit several skills, some of which they had not yet acquired or had difficulty applying, so they required close guidance from the teacher. Regarding the basic skills for teaching geometry using a compass and straightedge, it can be said that in order for the teacher to teach the material optimally, he should adapt creative teaching methods in class, be able to impart to students the various skills required to work with a compass and straightedge and assist them to develop them. The study also showed that there is a gap between the level of knowledge the teacher thinks the students have, and the level of knowledge that they really have. Considering this, the conclusions of the present study are that when teacher plans a geometry lesson using a compass and straightedge, he should first examine the level of the students’ knowledge, which necessary skills they have already adapted and which they have not yet acquired. In addition, geometry teachers need to acquire tools that can help them identify the difficulties that students deal with and fostering the students’ skills, so that they can adapt the teaching material to their level.

Limitations

This study examined a limited number of about 50 students from only two sixth grade classes, and therefore the results of this study cannot be generalized. The conclusions of the present study were based on one study unit thought to students in a limited time format. In addition, this study focused only on examining the impact of geometric constructions using a compass and straightedge on students’ attitudes, achievements, and motivation for studying. No additional teaching methods have been examined for the purpose of this comparison.

Recommendations

In further research it is recommended to examine how the issue of teaching geometry using a compass and straightedge in an elementary school is expressed from the point of view of teachers. It will be interesting to make a comparison between the students’ perceptions and those of geometry teachers. Furthermore, since gaps have been discovered between the level of knowledge that students possess in geometry, and the level of knowledge that the teacher thinks students possess, it is recommended to examine the reasons why these gaps exist, and how to reduce these gaps.

Acknowledgements

This article is based on the graduate work by (Taha – Heeb, 2021) under the supervision of the author.

References


Author Information

Khayriah Hasan Massarwe

http://orcid.org/0000-0003-1447-0321

Department of Mathematics

The Arab Academic College for Education in Israel - Haifa, Israel

Contact e-mail: massarwe@arabcol.ac.il